

Thinking Recursively

Part II

Outline for Today

- ***The Recursive Leap of Faith***
 - On trusting the contract.
- ***Enumerating Subsets***
 - A classic combinatorial problem.
- ***Decision Trees***
 - Generating all solutions to a problem.

Some Quick Refreshers

Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};  
cout << (mySet + 4) << endl; // Line A  
cout << (mySet - 3) << endl; // Line B
```

Answer at

<https://cs106b.stanford.edu/pollev>

Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};
```

```
cout << (mySet + 4) << endl; // Line A
```

```
cout << (mySet - 3) << endl; // Line B
```

```
{1, 2, 3}
```

```
Set<int> mySet
```

Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};  
cout << (mySet + 4) << endl; // Line A  
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

Set<int> mySet

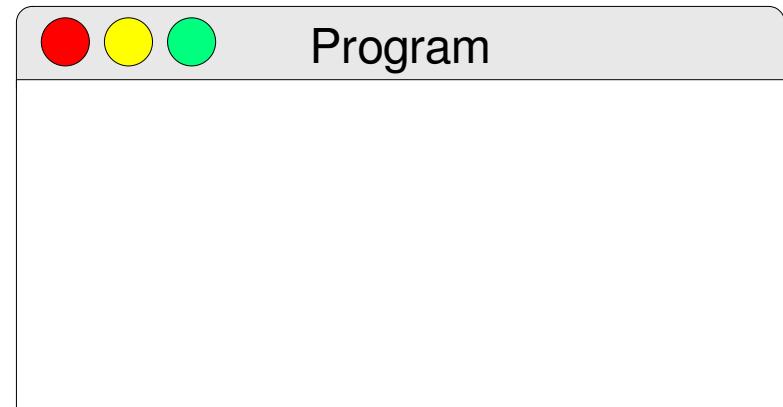
Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};  
cout << (mySet + 4) << endl; // Line A  
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

Set<int> mySet



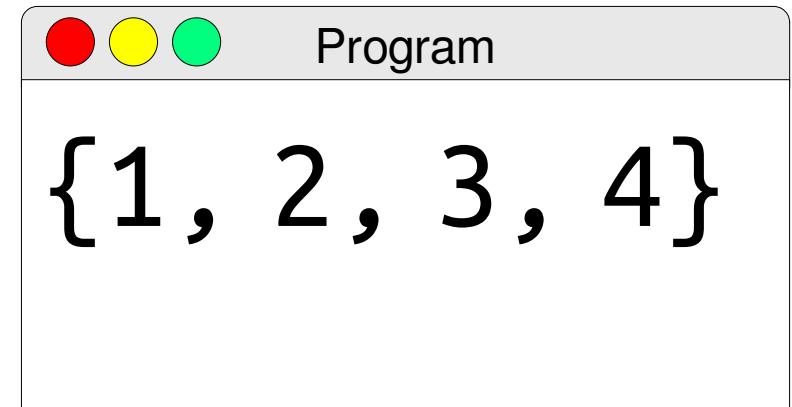
Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};  
cout << (mySet + 4) << endl; // Line A  
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

Set<int> mySet



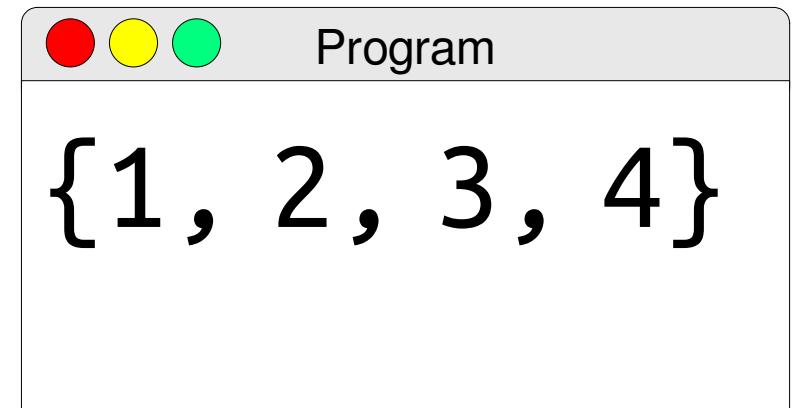
Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};  
cout << (mySet + 4) << endl; // Line A  
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

Set<int> mySet



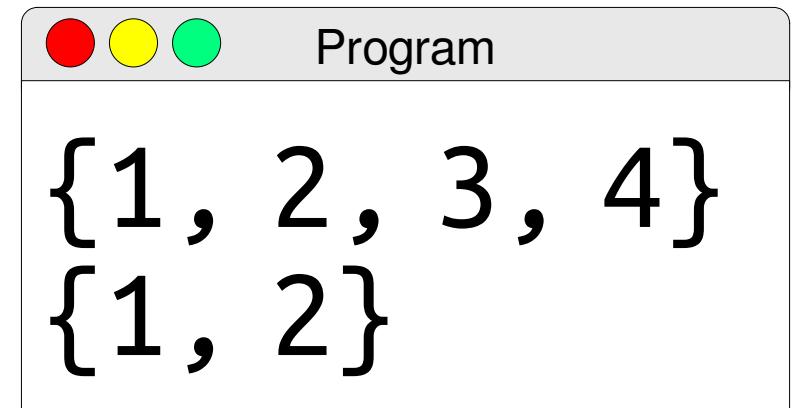
Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};  
cout << (mySet + 4) << endl; // Line A  
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

Set<int> mySet



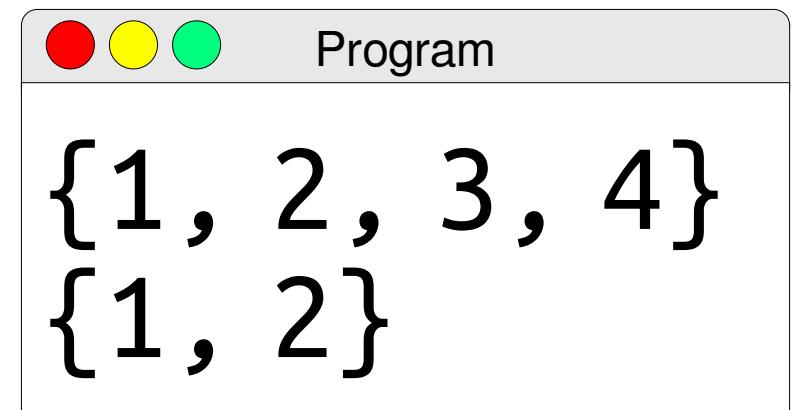
Set Refresher

- What's printed at Line A and Line B?

```
Set<int> mySet = {1, 2, 3};  
cout << (mySet + 4) << endl; // Line A  
cout << (mySet - 3) << endl; // Line B
```

{1, 2, 3}

Set<int> mySet



Recursion Refresher

- What does this code print?

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

```
squigglebah(2);
```

Answer at

<https://cs106b.stanford.edu/pollev>

squigglebah(2);

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n

S

```
• 11/16(2)  
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

1

int n

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

1

int n

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

1

int n

S

```
void squigglebah(int n) {
```

2

```
void squigglebah(int n) {
```

1

```
void squigglebah(int n) {
```

0

```
if (n != 0) {
```

int n

```
squigglebah(n - 1);
```

```
cout << n << endl;
```

```
}
```

```
}
```

S

```
void squigglebah(int n) {
```

2

```
    if (n > 0) {
```

1

```
        void squigglebah(int n) {
```

0

```
            if (n != 0) {
```

int n

```
                squigglebah(n - 1);
```

```
                cout << n << endl;
```

```
            }
```

```
        }
```

S

```
void squigglebah(int n) {
```

2

```
void squigglebah(int n) {
```

1

```
void squigglebah(int n) {
```

0

```
if (n != 0) {
```

int n

```
squigglebah(n - 1);
```

```
cout << n << endl;
```

```
}
```

```
}
```

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

1

int n

S

```
void squigglebah(int n) {
```

```
    if (n > 0) {
```

```
        squigglebah(n - 1);
```

```
        cout << n << endl;
```

```
}
```

2

1

int n

S

```
void squigglebah(int n) {
```

```
    if (n > 0) {
        squigglebah(n - 1);
        cout << n << endl;
    }
}
```

2

1

int n



Program

S

```
void squigglebah(int n) {  
    if (n > 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

1

int n



Program

1

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

1

int n



Program

1

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n



Program

1

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n



Program

1

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n



Program

1
2

S

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

2

int n

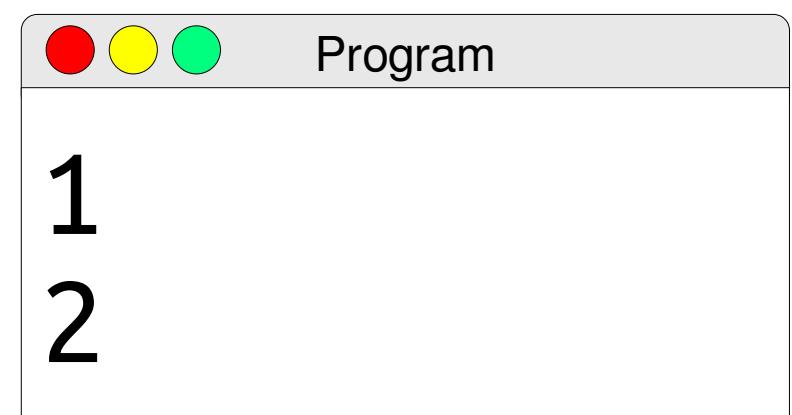


Program

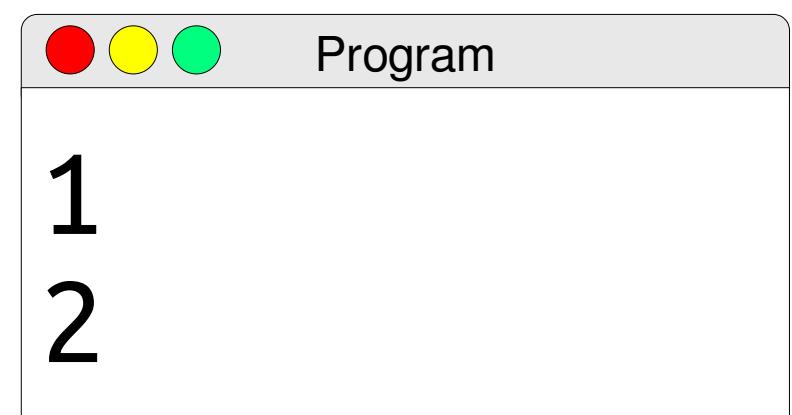
1

2

```
squigglebah(2);
```

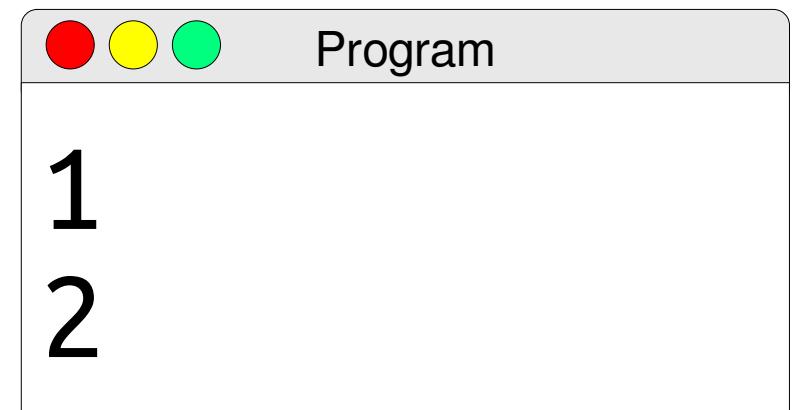


```
squigglebah(2);
```



```
squigglebah(2);
```

```
void squigglebah(int n) {  
    if (n != 0) {  
        squigglebah(n - 1);  
        cout << n << endl;  
    }  
}
```

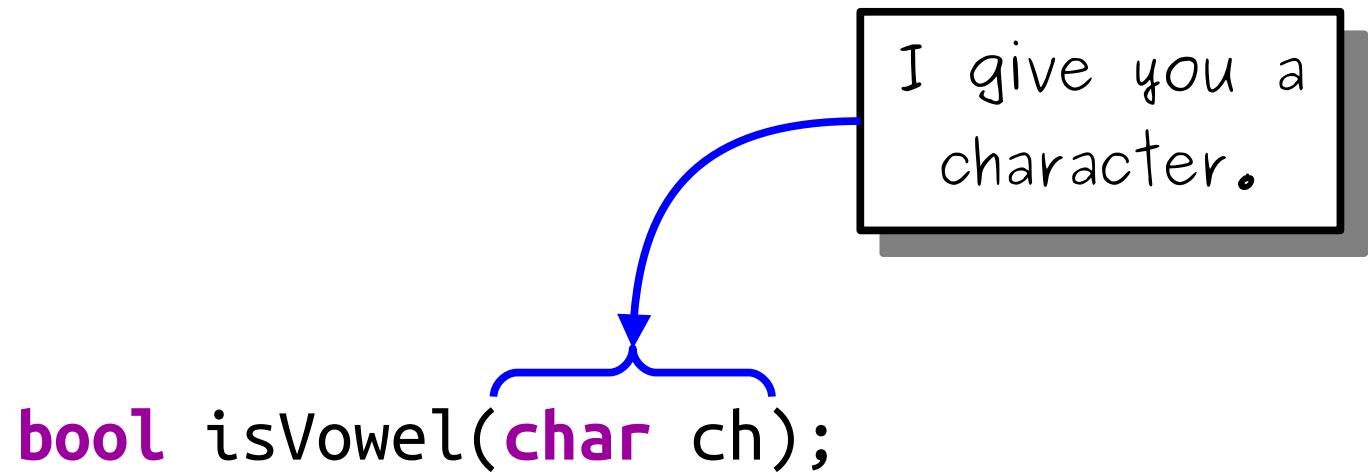


The Recursive Leap of Faith

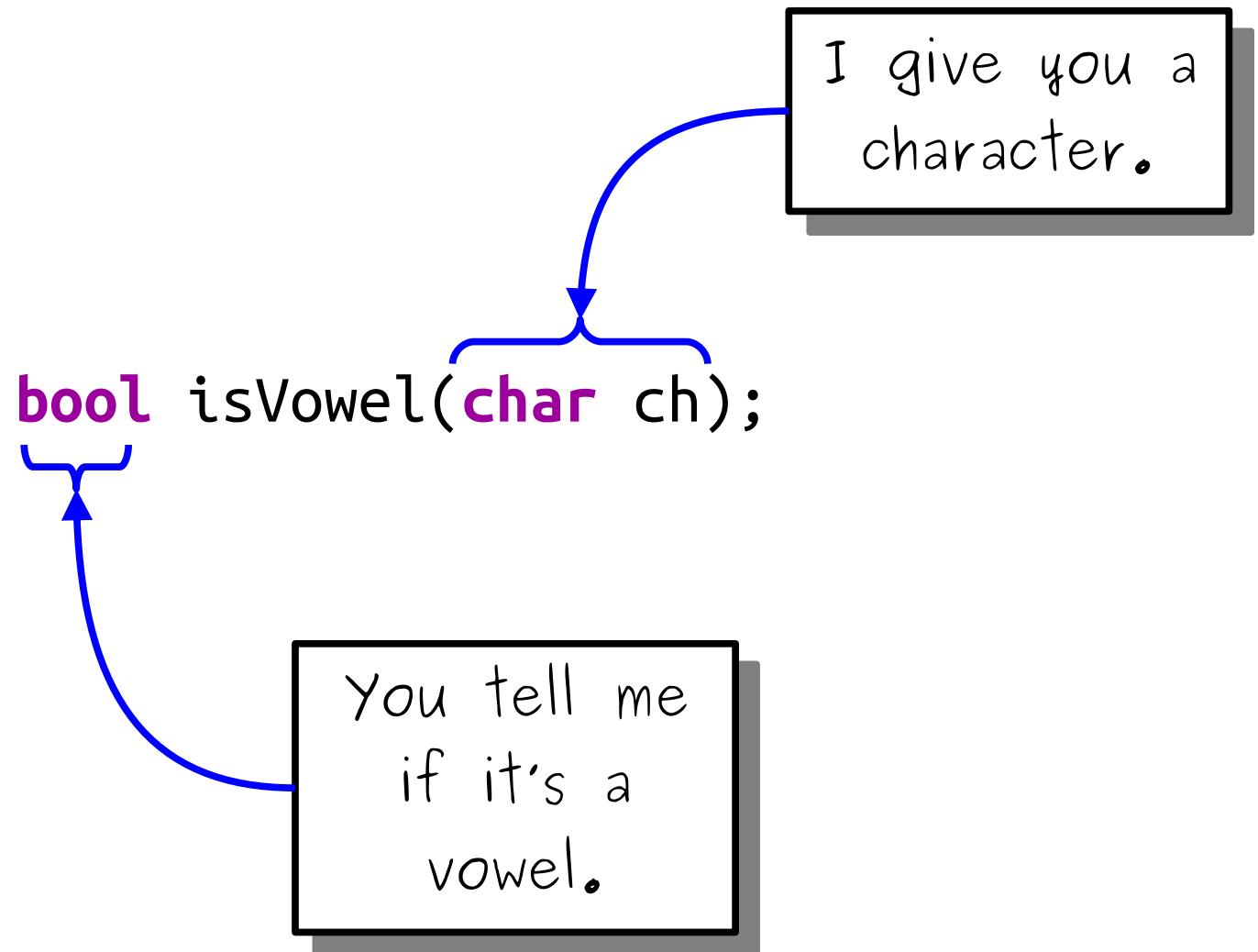
The Contract

```
bool isVowel(char ch);
```

The Contract



The Contract



The Contract

```
bool isVowel(char ch) {  
    ch = toLowerCase(ch);  
    return ch == 'a' ||  
           ch == 'e' ||  
           ch == 'i' ||  
           ch == 'o' ||  
           ch == 'u';  
}
```

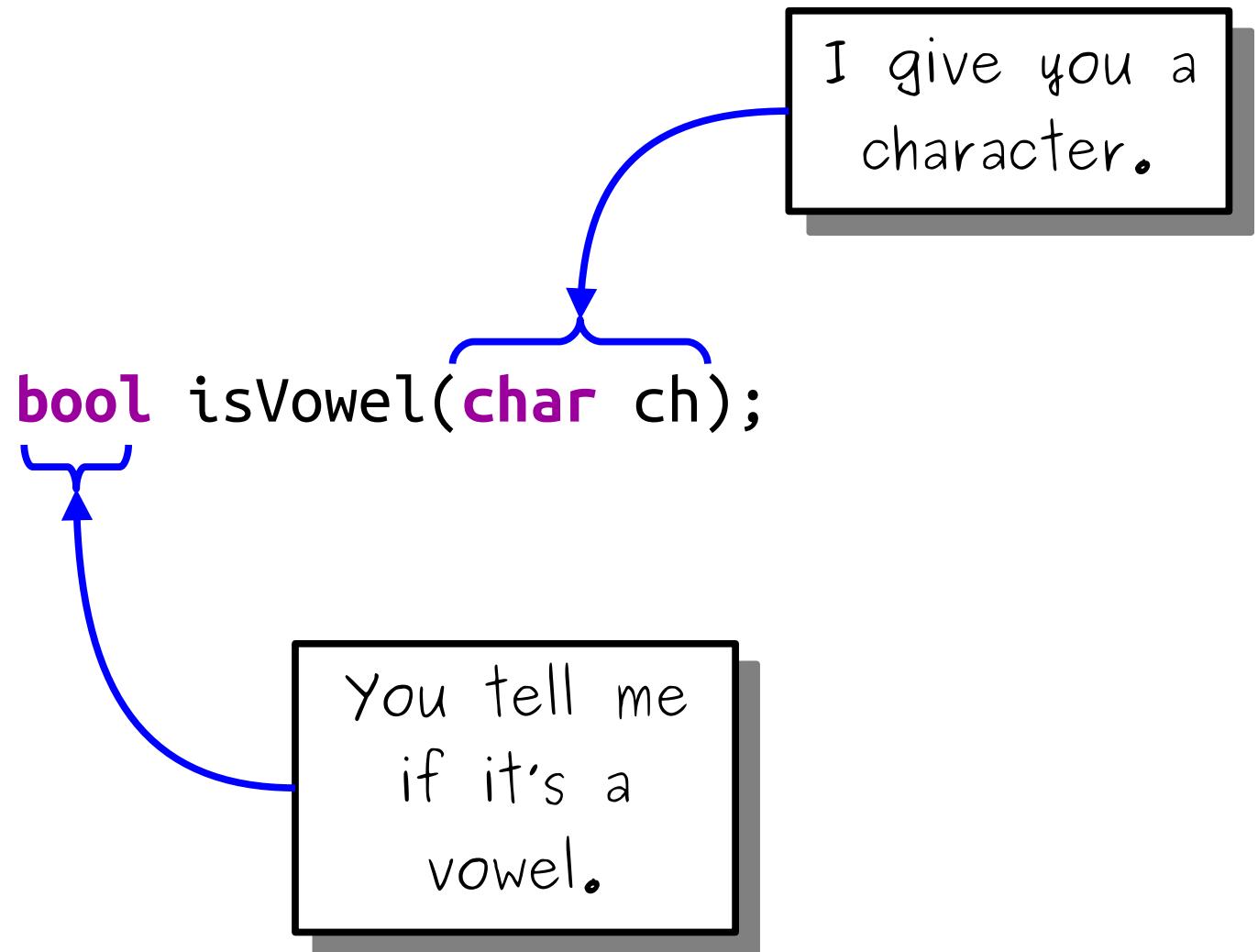
The Contract

```
bool isVowel(char ch) {  
    switch(ch) {  
        case 'A': case 'a':  
        case 'E': case 'e':  
        case 'I': case 'i':  
        case 'O': case 'o':  
        case 'U': case 'u':  
            return true;  
        default:  
            return false;  
    }  
}
```

The Contract

```
bool isVowel(char ch) {  
    ch = tolower(ch);  
    return string("aeiou").find(ch) != string::npos;  
}
```

The Contract



The Contract

The Contract

```
bool hasConsecutiveVowels(const string& str);
```

The Contract

I give you a
string.

bool hasConsecutiveVowels(**const** string& str);

The Contract

I give you a string.

```
bool hasConsecutiveVowels(const string& str);
```

You tell me if it has two or more consecutive letters that are vowels.

Trusting the Contract

```
bool isVowel(char ch);
```

```
bool hasConsecutiveVowels(const string& str) {
```

```
}
```

Trusting the Contract

```
bool isVowel(char ch);  
  
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
        }  
    }
```

Trusting the Contract

```
bool isVowel(char ch);  
  
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
        if (str[i - 1] is a vowel && str[i] is a vowel) {  
            return true;  
        }  
    }  
}
```

Trusting the Contract

```
bool isVowel(char ch);

bool hasConsecutiveVowels(const string& str) {
    for (int i = 1; i < str.length(); i++) {
        if (str[i - 1] is a vowel && str[i] is a vowel) {
            return true;
        }
    }
    return false;
}
```

Trusting the Contract

```
bool isVowel(char ch);
```

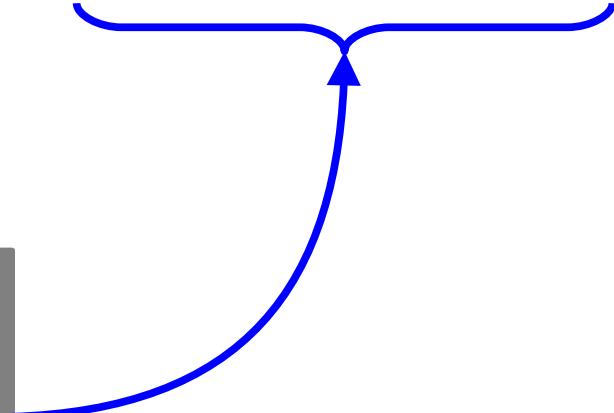
```
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
        if (isVowel(str[i - 1]) && isVowel(str[i])) {  
            return true;  
        }  
    }  
    return false;  
}
```

Trusting the Contract

```
bool isVowel(char ch);
```

```
bool hasConsecutiveVowels(const string& str) {  
    for (int i = 1; i < str.length(); i++) {  
        if (isVowel(str[i - 1]) && isVowel(str[i])) {  
            return true;  
        }  
    }  
    return false;  
}
```

It doesn't matter how
isVowel is implemented.
We just trust that it
works.

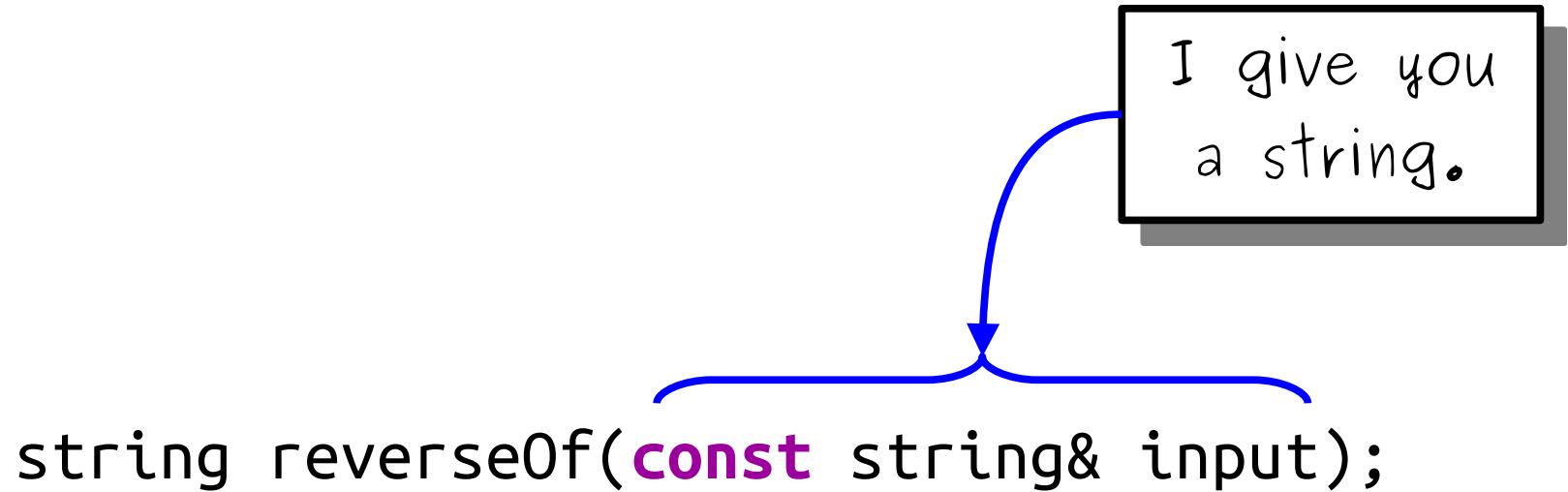


The Contract

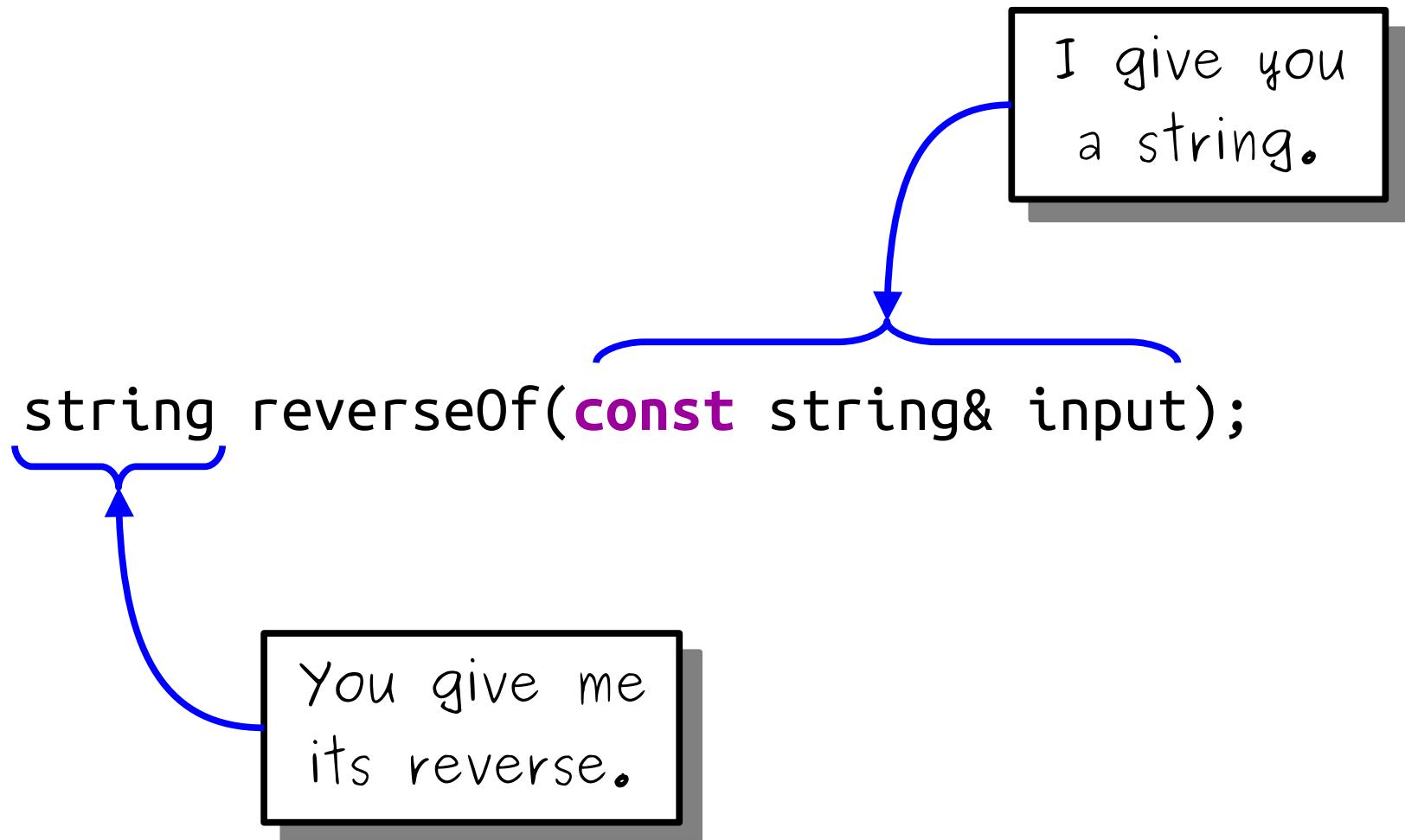
The Contract

```
string reverse0f(const string& input);
```

The Contract



The Contract



Trusting the Contract

```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
  
}
```

Trusting the Contract

```
string reverseOf(const string& input);

string reverseOf(const string& input) {
    if (input == "") {

    } else {

    }
}
```

Trusting the Contract

```
string reverseOf(const string& input);

string reverseOf(const string& input) {
    if (input == "") {
        return "";
    } else {

    }
}
```

Trusting the Contract

```
string reverseOf(const string& input);

string reverseOf(const string& input) {
    if (input == "") {
        return "";
    } else {
        return the reverse of input.substr(1) + input[0];
    }
}
```

Trusting the Contract

```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
    if (input == "") {  
        return "";  
    } else {  
        return reverseOf(input.substr(1)) + input[0];  
    }  
}
```

Trusting the Contract

```
string reverseOf(const string& input);

string reverseOf(const string& input) {
    if (input == "") {
        return "";
    } else {
        return reverseOf(input.substr(1)) + input[0];
    }
}
```

Trusting the Contract

```
string reverseOf(const string& input);  
  
string reverseOf(const string& input) {  
    if (input == "") {  
        return "";  
    } else {  
        return reverseOf(input.substr(1)) + input[0];  
    }  
}
```

It doesn't matter how reverseOf reverses the string. It just matters that it does.

The Contract

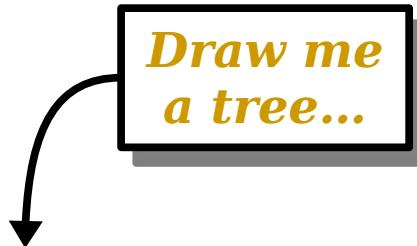
The Contract

```
void drawTree(double x, double y,  
             double height,  
             double angle,  
             int order);
```

The Contract

```
void drawTree(double x, double y,  
             double height,  
             double angle,  
             int order);
```

The Contract



```
void drawTree(double x, double y,  
             double height,  
             double angle,  
             int order);
```

The Contract

The diagram illustrates a software contract. At the top left, a callout box contains the text "Draw me a tree...". An arrow points from this box down to the first parameter of the `drawTree` function. At the top right, another callout box contains the text "... at this position ...". An arrow points from this box down to the second parameter of the `drawTree` function.

```
void drawTree(double x, double y,  
             double height,  
             double angle,  
             int order);
```

The Contract

```
void drawTree(double x, double y,  
             double height,  
             double angle,  
             int order);
```

The diagram illustrates the contract for the `drawTree` function. It features three callout boxes with arrows pointing to specific parameters:

- An arrow points from the first parameter (`x`) to a box containing the text *Draw me a tree...* in yellow.
- An arrow points from the second parameter (`y`) to a box containing the text *... at this position ...* in teal.
- An arrow points from the third parameter (`height`) to a box containing the text *... that's this big ...* in purple.

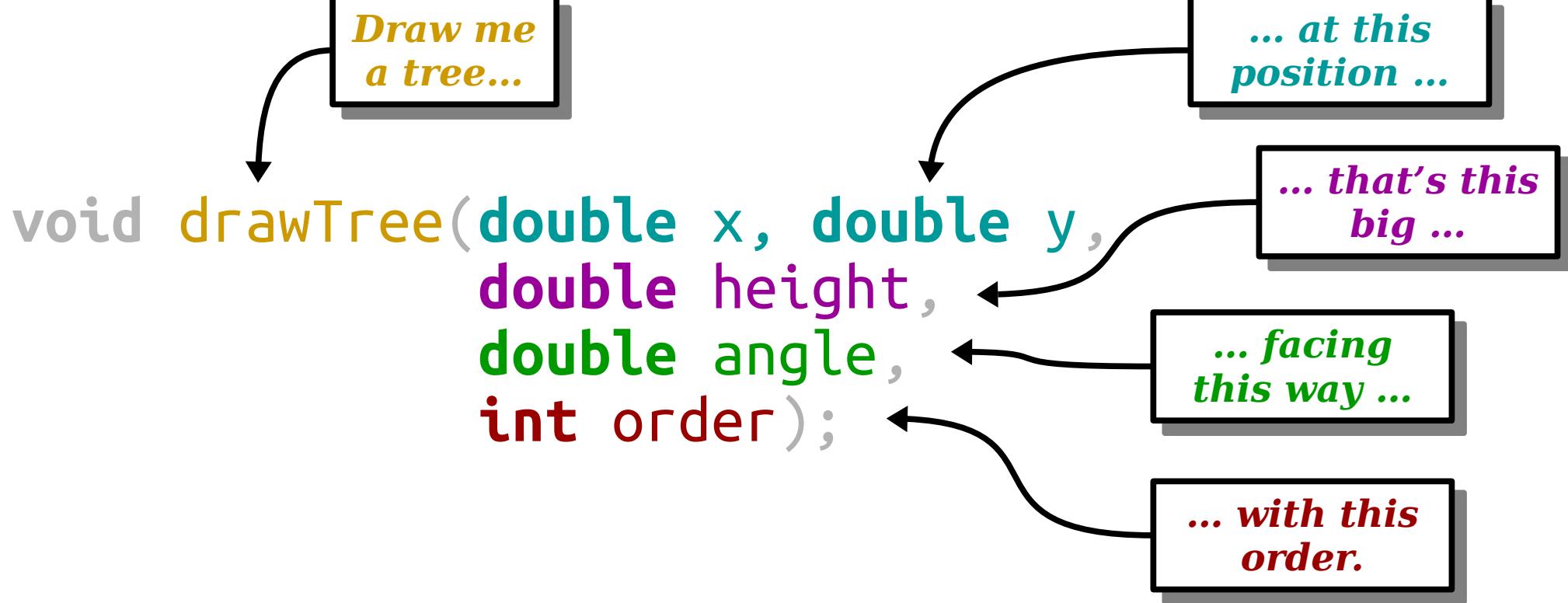
The Contract

```
void drawTree(double x, double y,  
             double height,  
             double angle,  
             int order);
```

The diagram illustrates the contract for the `drawTree` function. It features four callout boxes with arrows pointing to specific parameters:

- An arrow points from the first parameter (`x`) to a box containing *Draw me a tree...* in yellow text.
- An arrow points from the second parameter (`y`) to a box containing *... at this position ...* in cyan text.
- An arrow points from the third parameter (`height`) to a box containing *... that's this big ...* in magenta text.
- An arrow points from the fourth parameter (`angle`) to a box containing *... facing this way ...* in green text.

The Contract



Trusting the Contract

```
void drawTree(double x, double y,  
             double height, double angle,  
             int order);  
  
void drawTree(double x, double y,  
             double height, double angle,  
             int order) {  
}  
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
             double height, double angle,  
             int order);
```

```
void drawTree(double x, double y,  
             double height, double angle,  
             int order) {  
    if (order == 0) return;  
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
             double height, double angle,  
             int order);  
  
void drawTree(double x, double y,  
             double height, double angle,  
             int order) {  
    if (order == 0) return;  
  
    GPoint endpoint = drawPolarLine(/* ... */);  
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
             double height, double angle,  
             int order);  
  
void drawTree(double x, double y,  
             double height, double angle,  
             int order) {  
    if (order == 0) return;  
  
    GPoint endpoint = drawPolarLine(/* ... */);  
  
    draw a tree angling to the left  
    draw a tree angling to the right  
}
```

Trusting the Contract

```
void drawTree(double x, double y,
              double height, double angle,
              int order);

void drawTree(double x, double y,
              double height, double angle,
              int order) {
    if (order == 0) return;

    GPoint endpoint = drawPolarLine(/* ... */);

    drawTree(/* ... */);
    drawTree(/* ... */);
}
```

Trusting the Contract

```
void drawTree(double x, double y,  
             double height, double angle,  
             int order);
```

```
void drawTree(double x, double y,  
             double height, double angle,  
             int order) {  
    if (order == 0) return;  
  
    GPoint endpoint = drawPolarLine(..., ...);
```

It doesn't matter how
drawTree draws a
tree. It just matters
that it does.

```
    drawTree(/* ... */); }  
    drawTree(/* ... */); }
```

The Recursive Leap of Faith

- When writing a recursive function, it helps to take a ***recursive leap of faith***.
- Before writing the function, answer these questions:
 - What does the function take in?
 - What does it return?
- Then, as you’re writing the function, trust that your recursive calls to the function just “work” without asking how.
- This can take some adjustment to get used to, but is a necessary skill for writing more complex recursive functions.

Time-Out for Announcements!

Recursive Drawing Contest

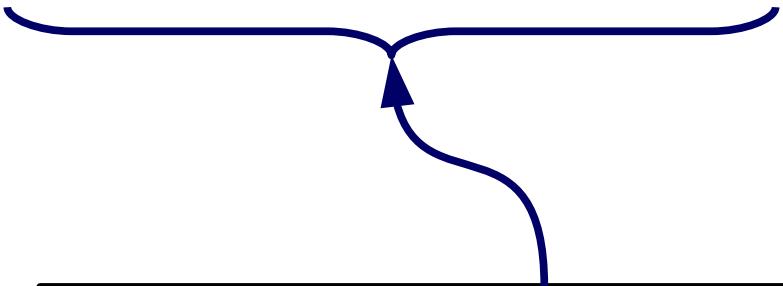
- We are holding a (purely optional, just for fun) Recursive Drawing contest!
- Visit <http://recursivedrawing.com/>, draw whatever you'd like, and post it to the EdStem thread for the contest.
- We'll award recursion-themed prizes to a small number of entries.
- Deadline to submit is Monday at 1:00PM Pacific.

Assignment 2

- Assignment 2 is due this Friday at 1:00PM.
 - If you're following our timetable, you'll have finished Rosetta Stone at this point and be midway through Rising Tides.
- Have questions?
 - Stop by the LaIR!
 - Ask on EdStem!
 - Visit our office hours!

Back to CS106B!

Recursive Enumeration



e·nu·mer·a·tion

noun

The act of mentioning a number of things one by one.

(Source: Oxford Languages, via Google)

Listing Subsets

- A set S is a **subset** of a set T when every element of S is an element of T .
- There are two subsets of $\{2\}$:

$\{ \}$ $\{2\}$

- There are four subsets of $\{2, 3\}$:

$\{ \}$ $\{2\}$ $\{3\}$ $\{2, 3\}$

- How many subsets are there of $\{2, 3, 4\}$?

Answer at

<https://cs106b.stanford.edu/pollev>

Listing Subsets

- A set S is a **subset** of a set T when every element of S is an element of T .

- There are two subsets of $\{2\}$:

$$\{ \} \quad \{2\}$$

- There are four subsets of $\{2, 3\}$:

$$\{ \} \quad \{2\} \quad \{3\} \quad \{2, 3\}$$

- How many subsets are there of $\{2, 3, 4\}$?

$$\begin{array}{c} \{ \} \\ \{2\} \quad \{3\} \quad \{4\} \\ \{2, 3\} \quad \{2, 4\} \quad \{3, 4\} \\ \{2, 3, 4\} \end{array}$$

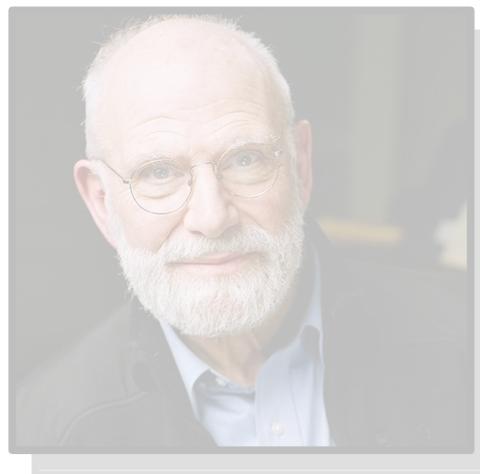
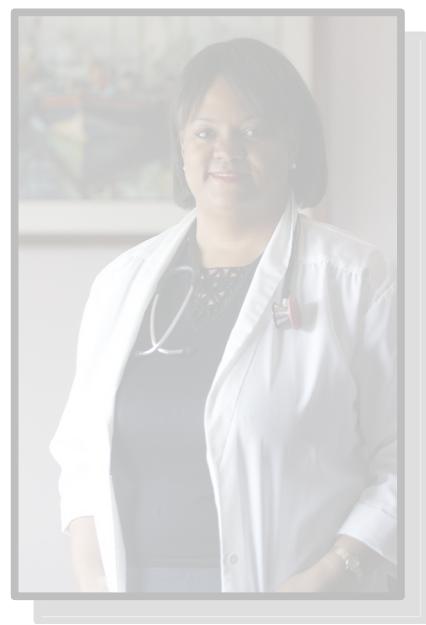
- The only subset of $\{ \}$ is $\{ \}$.

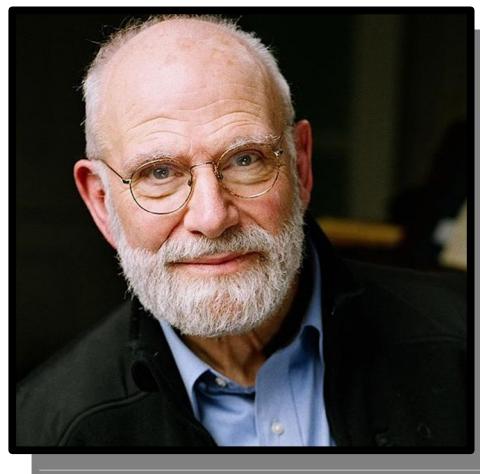
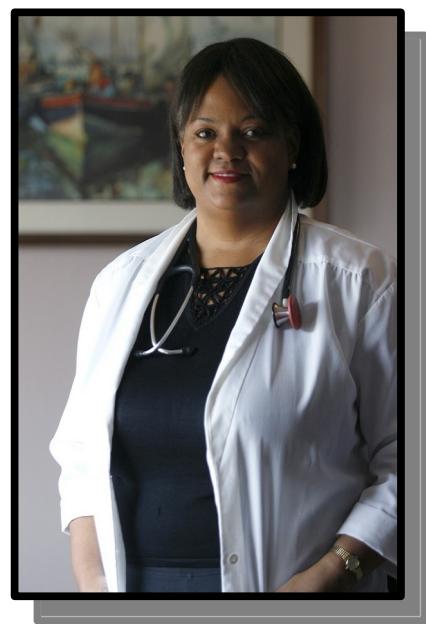


You need to send an emergency team of doctors to an area.

You know which doctors you have available to send.

List all the possible teams you can make from your list of all the doctors.





1

2

3

4

{ 1 , 2 , 3 , 4 }

{ 1, 2, 3, 4 }

{	1			
{	1	,		4
{	1	,	3	
{	1	,	3	, 4
{	1	,	2	
{	1	,	2	, 4
{	1	,	2	, 3
{	1	,	2	, 3 , 4

{ 1, 2, 3, 4 }

These are all the subsets of
 $\{ 2, 3, 4 \}$.

{ 1, 2, 3, 4 }

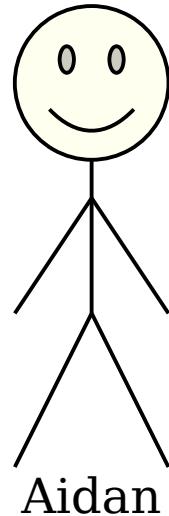
These are all the subsets of
 $\{ 2, 3, 4 \}$ with 1 inserted into them.

2 , 3 , 4 }
2 , 3 }
2 , 4 }

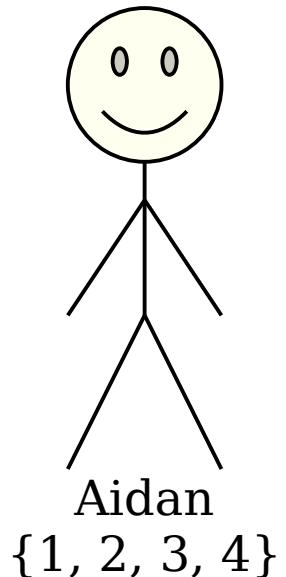
{	1		}
{	1 ,		4
{	1 ,	3	
{	1 ,	3 , 4	
{	1 , 2		
{	1 , 2 ,		4
{	1 , 2 , 3		
{	1 , 2 , 3 , 4		

Aidans List Subsets

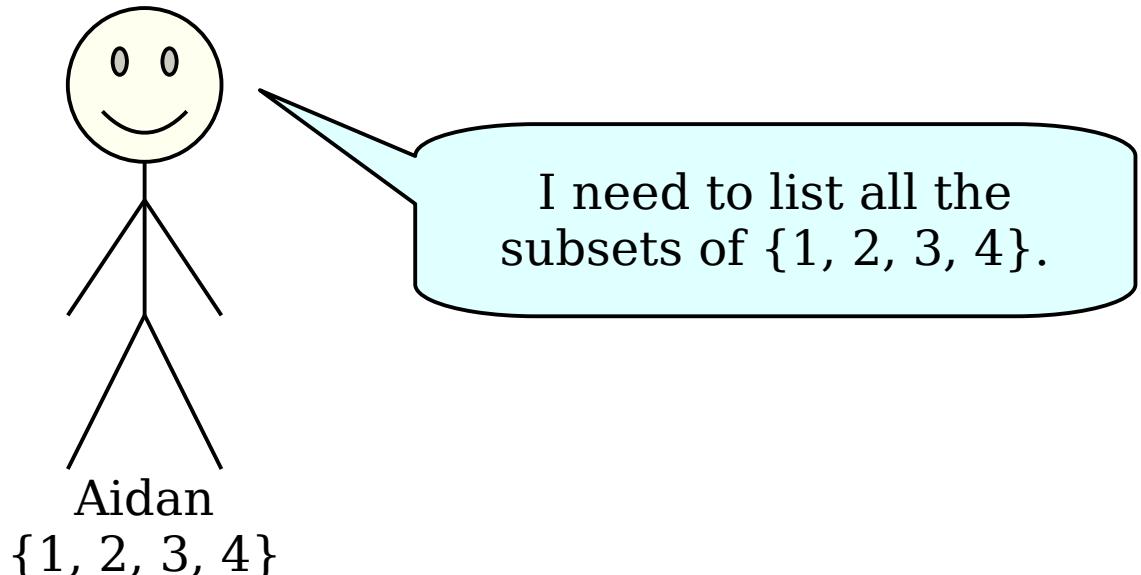
Aidans List Subsets



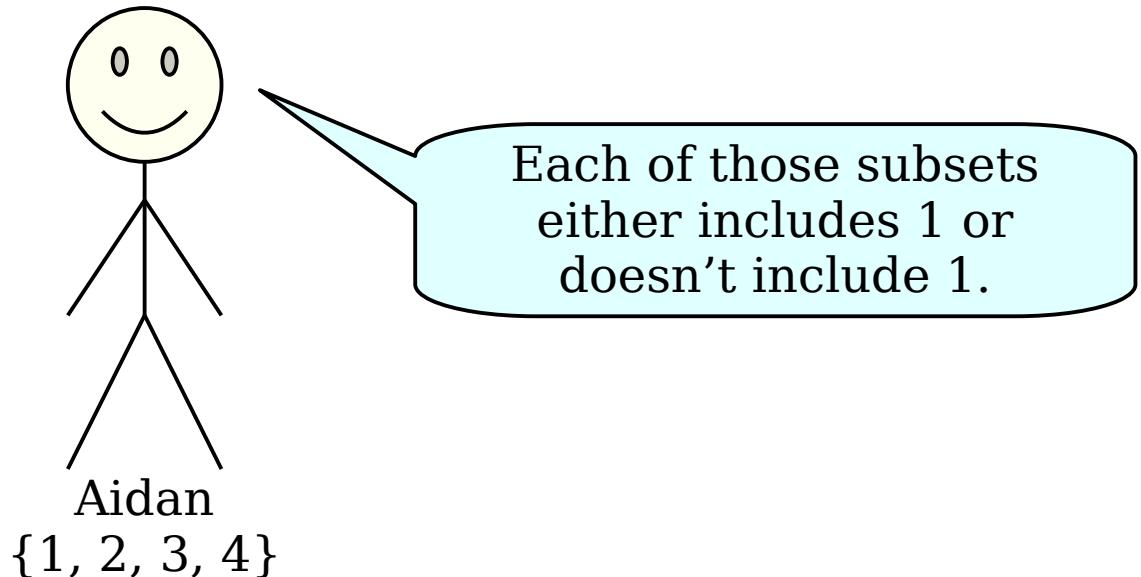
Aidans List Subsets



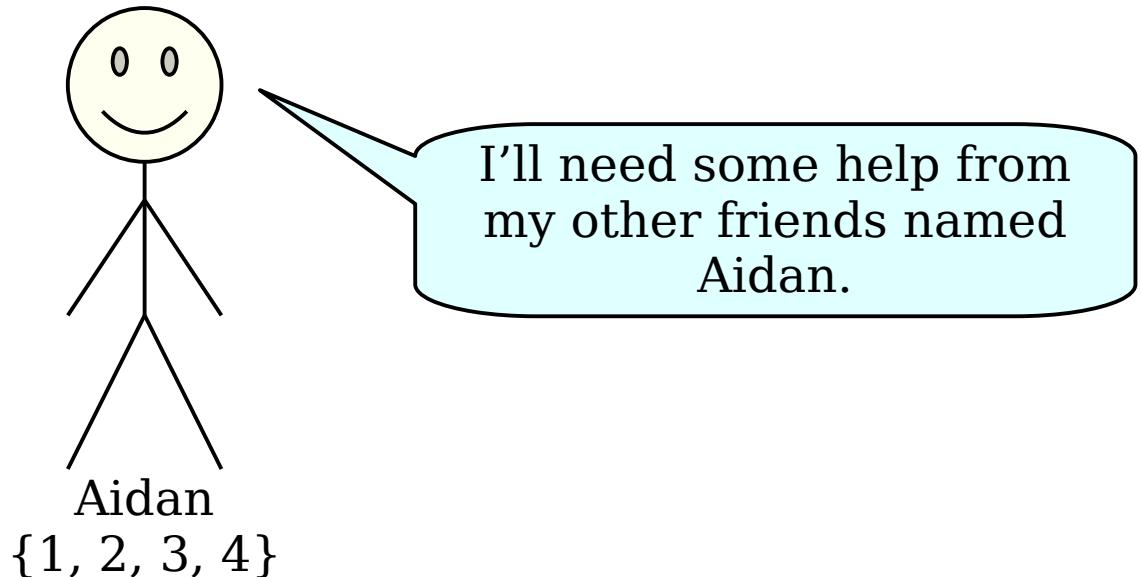
Aidans List Subsets



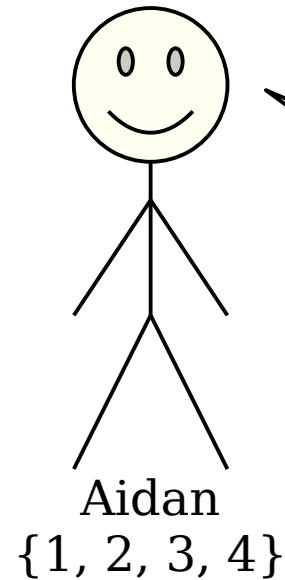
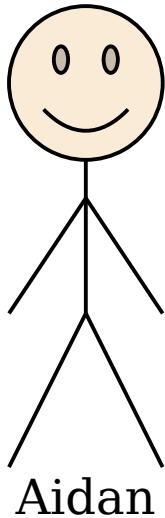
Aidans List Subsets



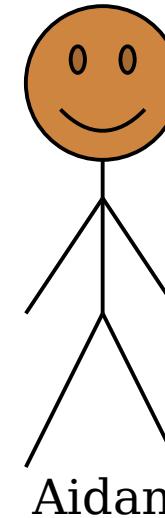
Aidans List Subsets



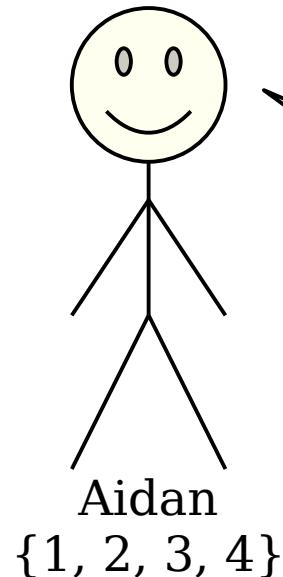
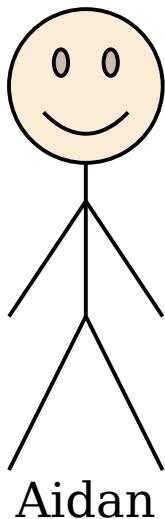
Aidans List Subsets



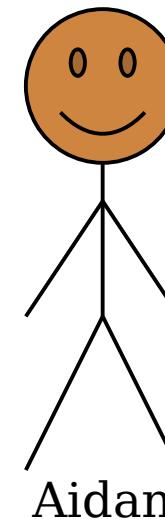
Can I get two more
Aidans here to help out?



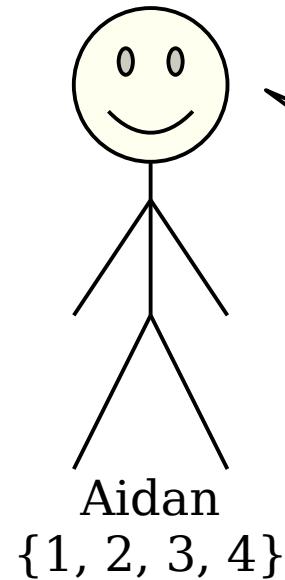
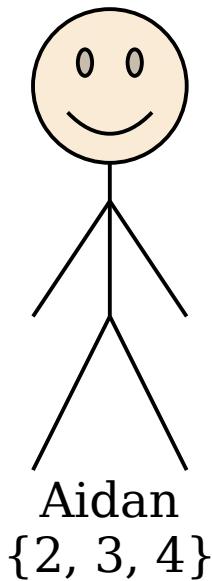
Aidans List Subsets



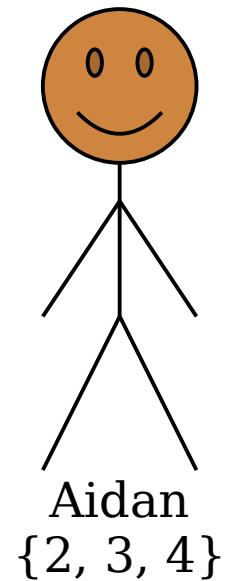
I need each of you to list off all the subsets of the set $\{2, 3, 4\}$.



Aidans List Subsets

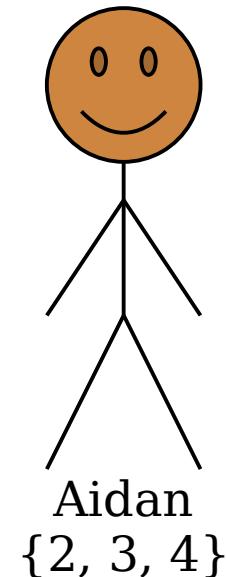
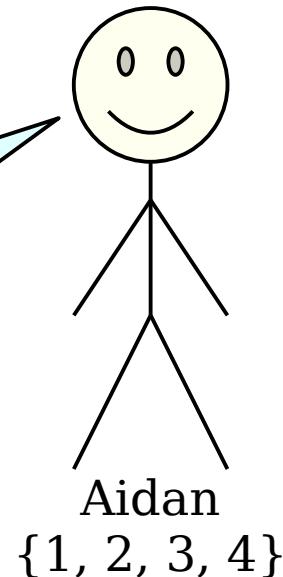
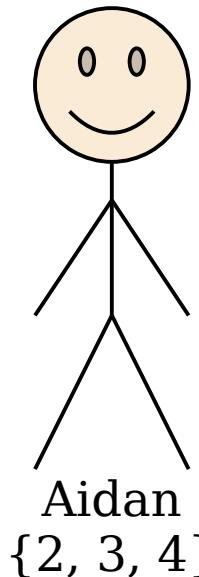


I need each of you to list off all the subsets of the set $\{2, 3, 4\}$.



Aidans List Subsets

Aidan to my left - as you list those subsets, insert a 1 into each.



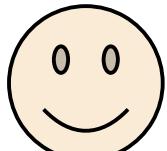
Aidans List Subsets

Aidan to my left - as you list those subsets, insert a 1 into each.



Aidan

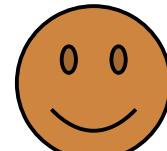
$\{1, 2, 3, 4\}$



Aidan

$\{2, 3, 4\}$

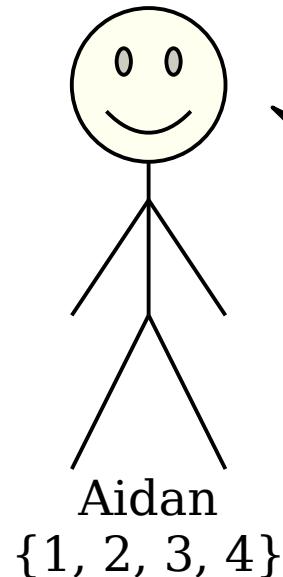
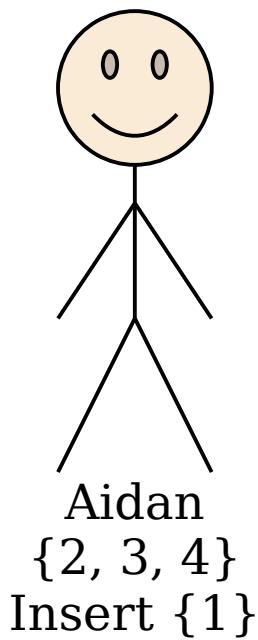
Insert $\{1\}$



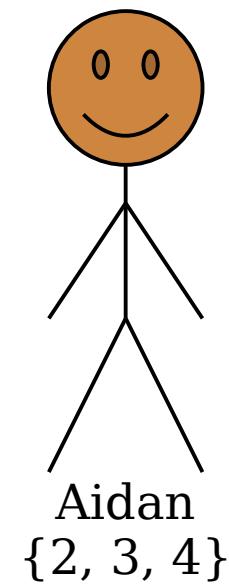
Aidan

$\{2, 3, 4\}$

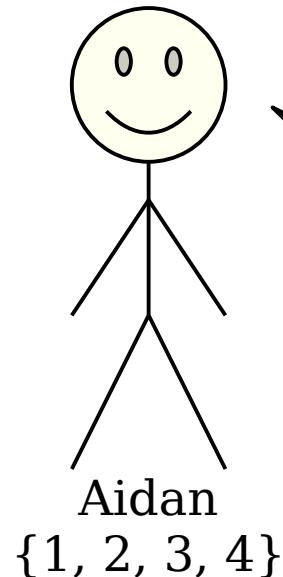
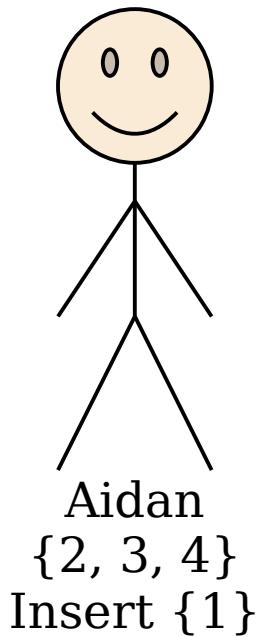
Aidans List Subsets



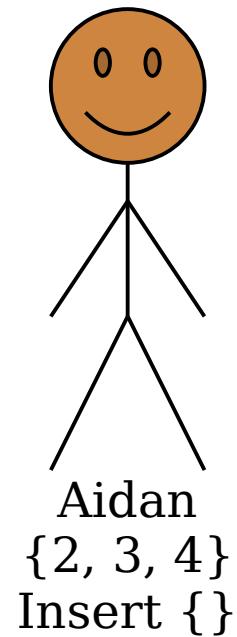
Aidan to my right – just list those subsets as-is.
Don't insert anything.



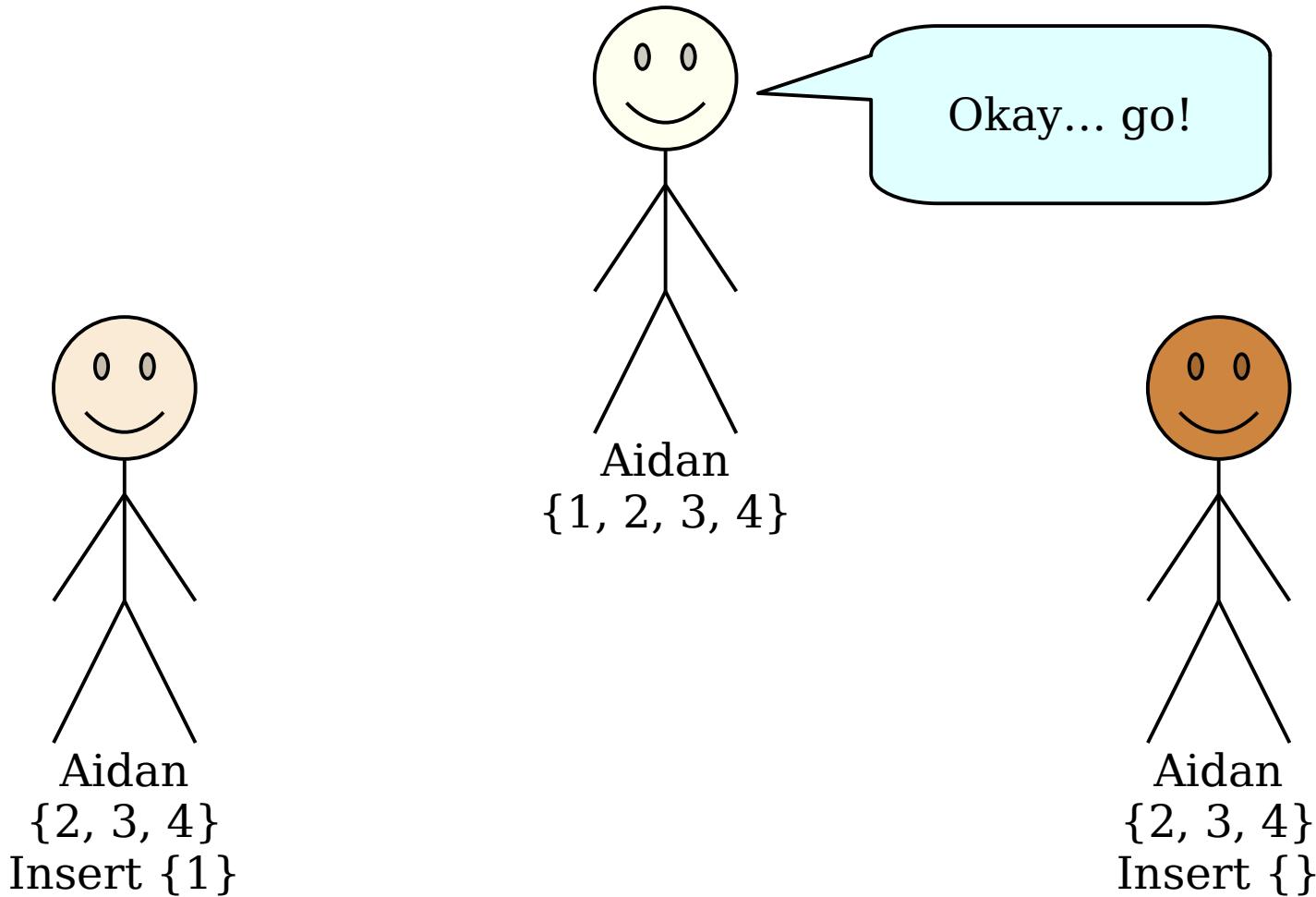
Aidans List Subsets



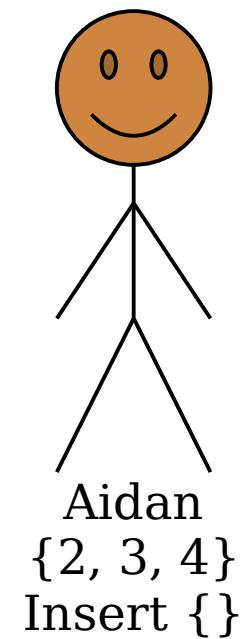
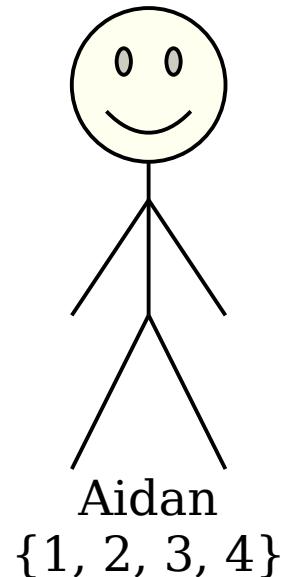
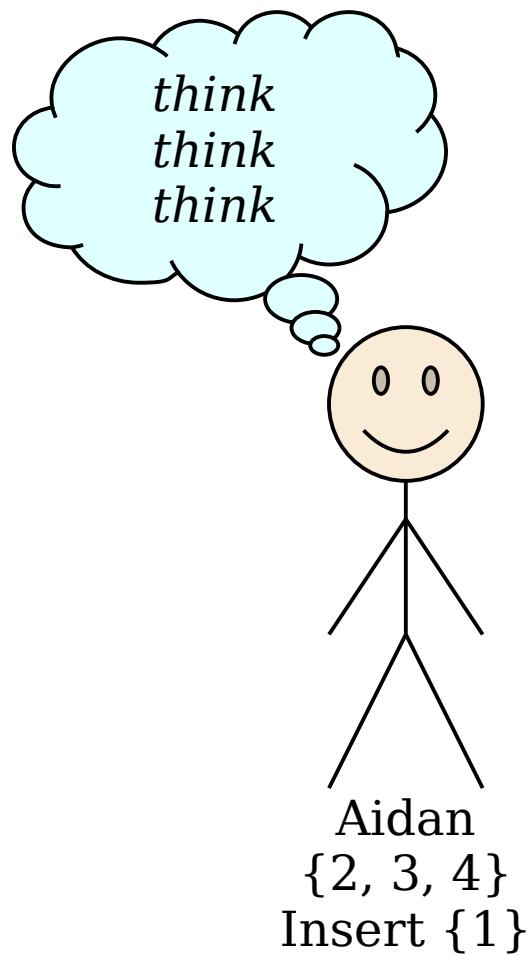
Aidan to my right – just list those subsets as-is.
Don't insert anything.



Aidans List Subsets

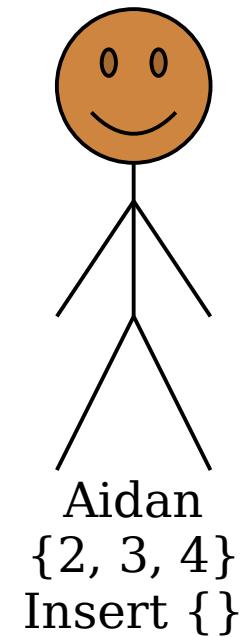
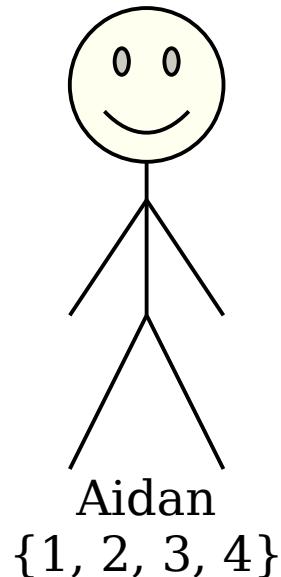
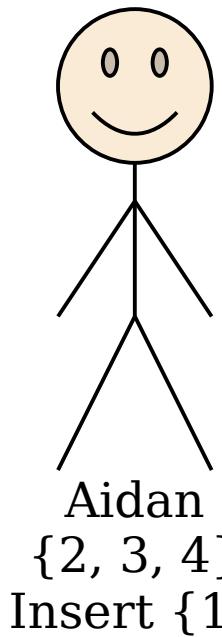


Aidans List Subsets



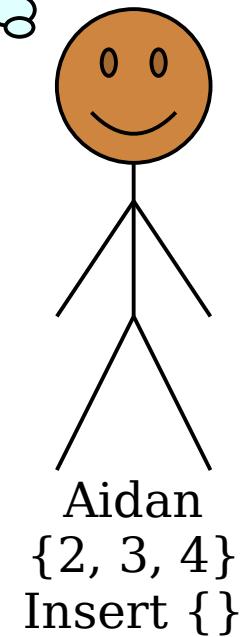
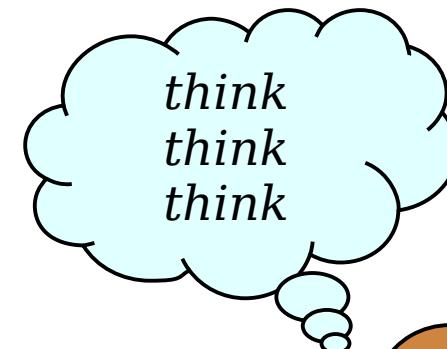
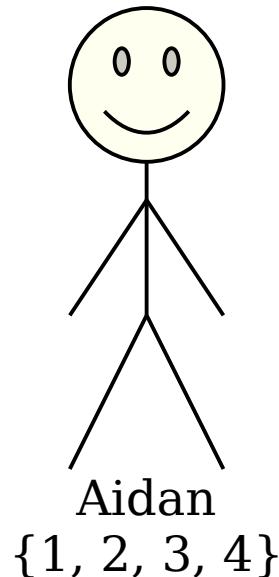
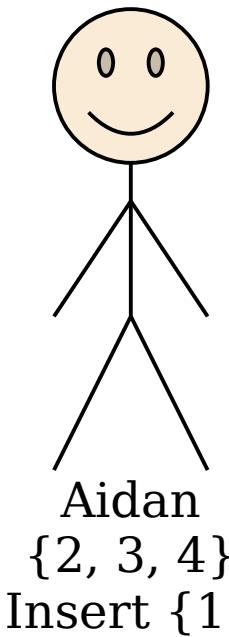
Aidans List Subsets

Here's what
you asked for!



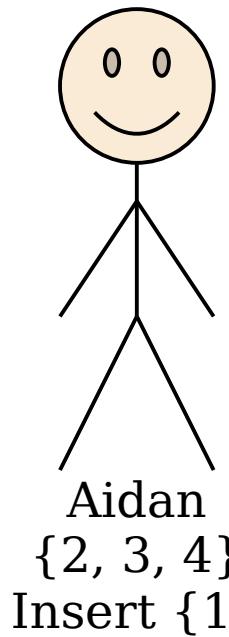
$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$

Aidans List Subsets

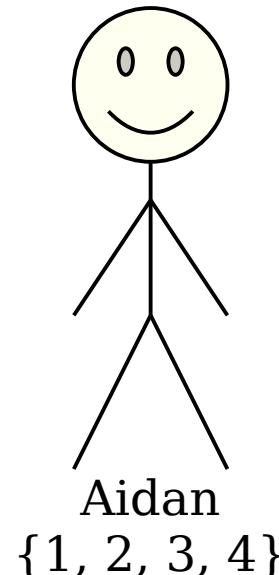


{1}	{1, 2}
{1, 4}	{1, 2, 4}
{1, 3}	{1, 2, 3}
{1, 3, 4}	{1, 2, 3, 4}

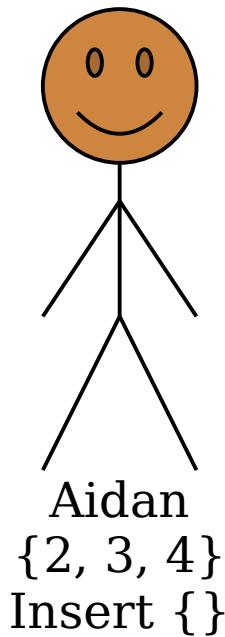
Aidans List Subsets



$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$



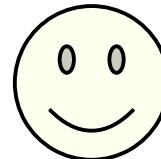
Here's what
you asked for!



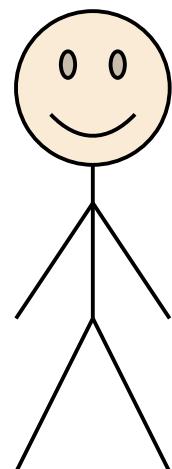
$\{\}$	$\{2\}$
$\{4\}$	$\{2, 4\}$
$\{3\}$	$\{2, 3\}$
$\{3, 4\}$	$\{2, 3, 4\}$

Aidans List Subsets

Thanks! You
guys are
great.

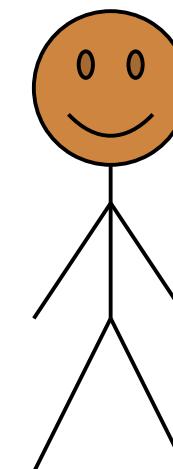


Aidan
 $\{1, 2, 3, 4\}$



Aidan
 $\{2, 3, 4\}$
Insert $\{1\}$

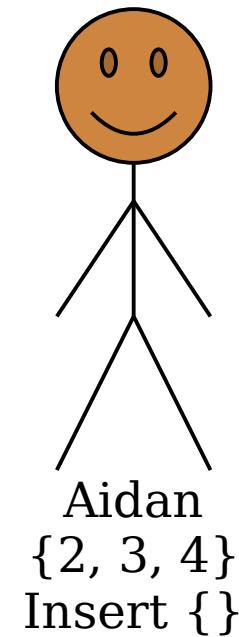
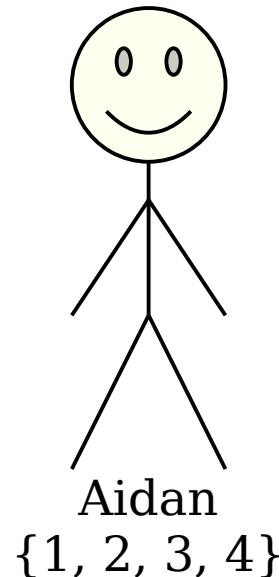
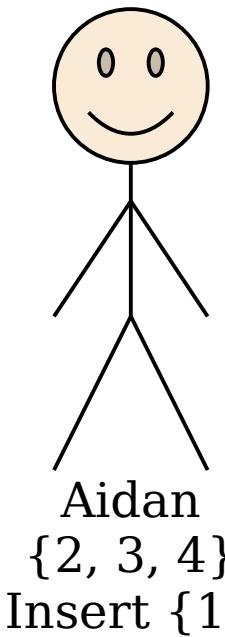
$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$



Aidan
 $\{2, 3, 4\}$
Insert $\{\}$

$\{\}$	$\{2\}$
$\{4\}$	$\{2, 4\}$
$\{3\}$	$\{2, 3\}$
$\{3, 4\}$	$\{2, 3, 4\}$

Aidans List Subsets

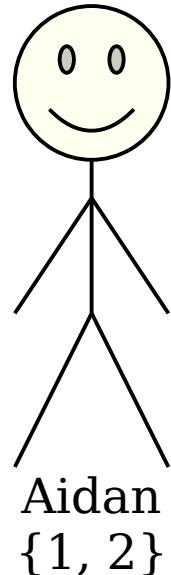


$\{1\}$	$\{1, 2\}$
$\{1, 4\}$	$\{1, 2, 4\}$
$\{1, 3\}$	$\{1, 2, 3\}$
$\{1, 3, 4\}$	$\{1, 2, 3, 4\}$

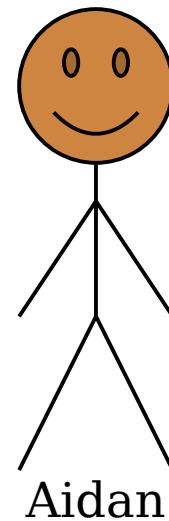
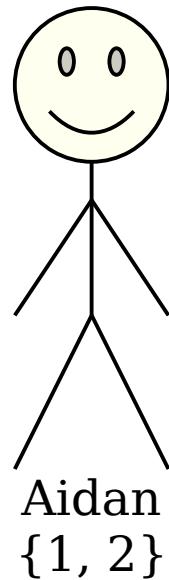
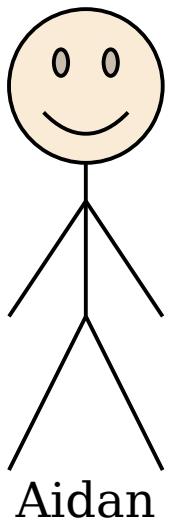
$\{\}$	$\{2\}$
$\{4\}$	$\{2, 4\}$
$\{3\}$	$\{2, 3\}$
$\{3, 4\}$	$\{2, 3, 4\}$

Thinking Recursively

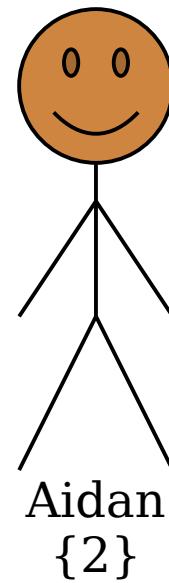
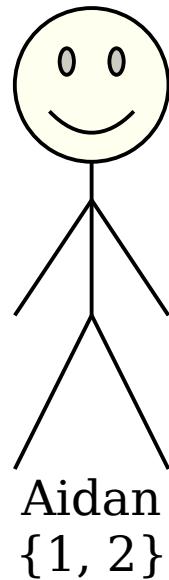
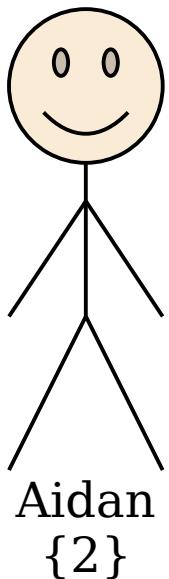
Aidans List Subsets



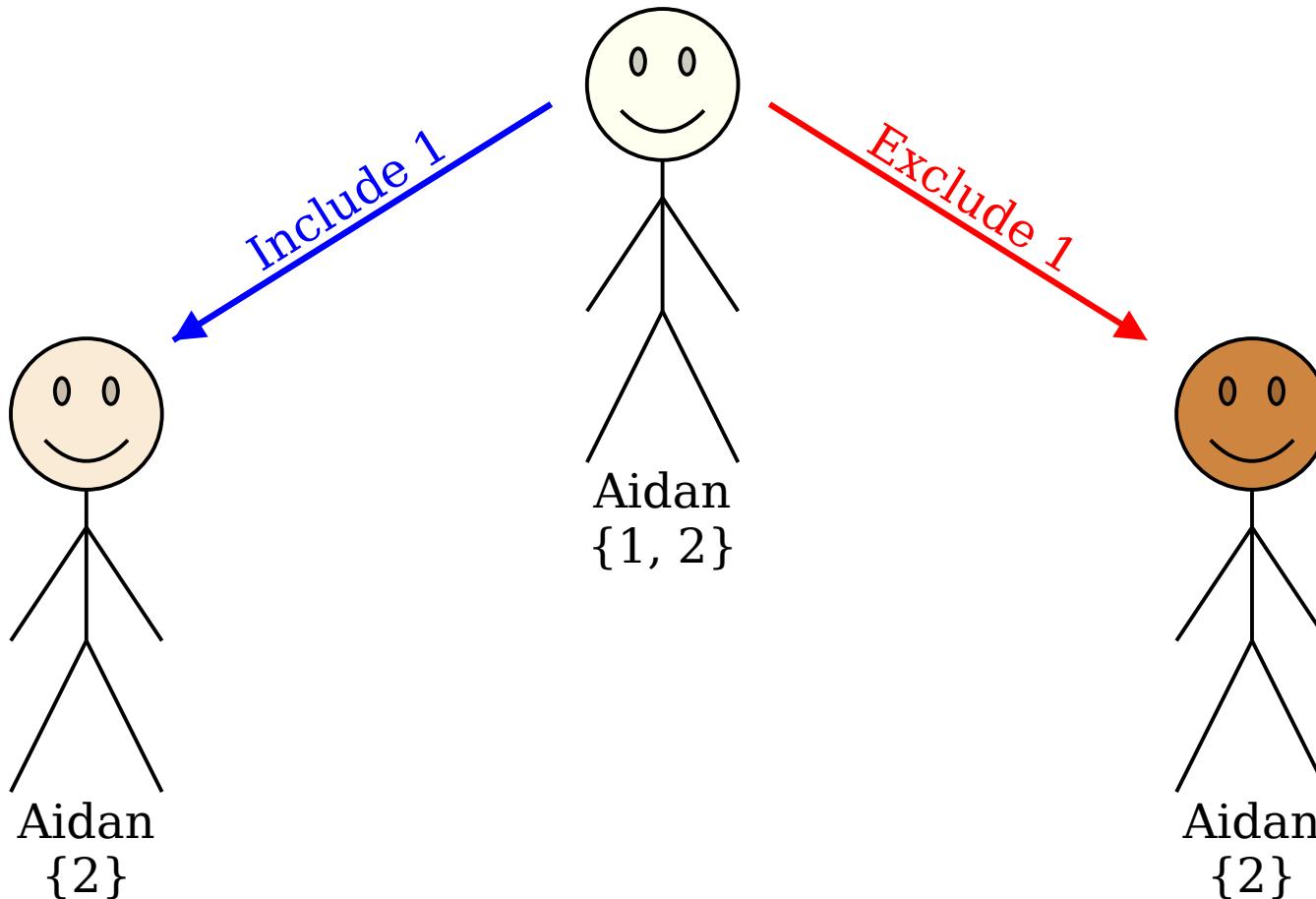
Aidans List Subsets



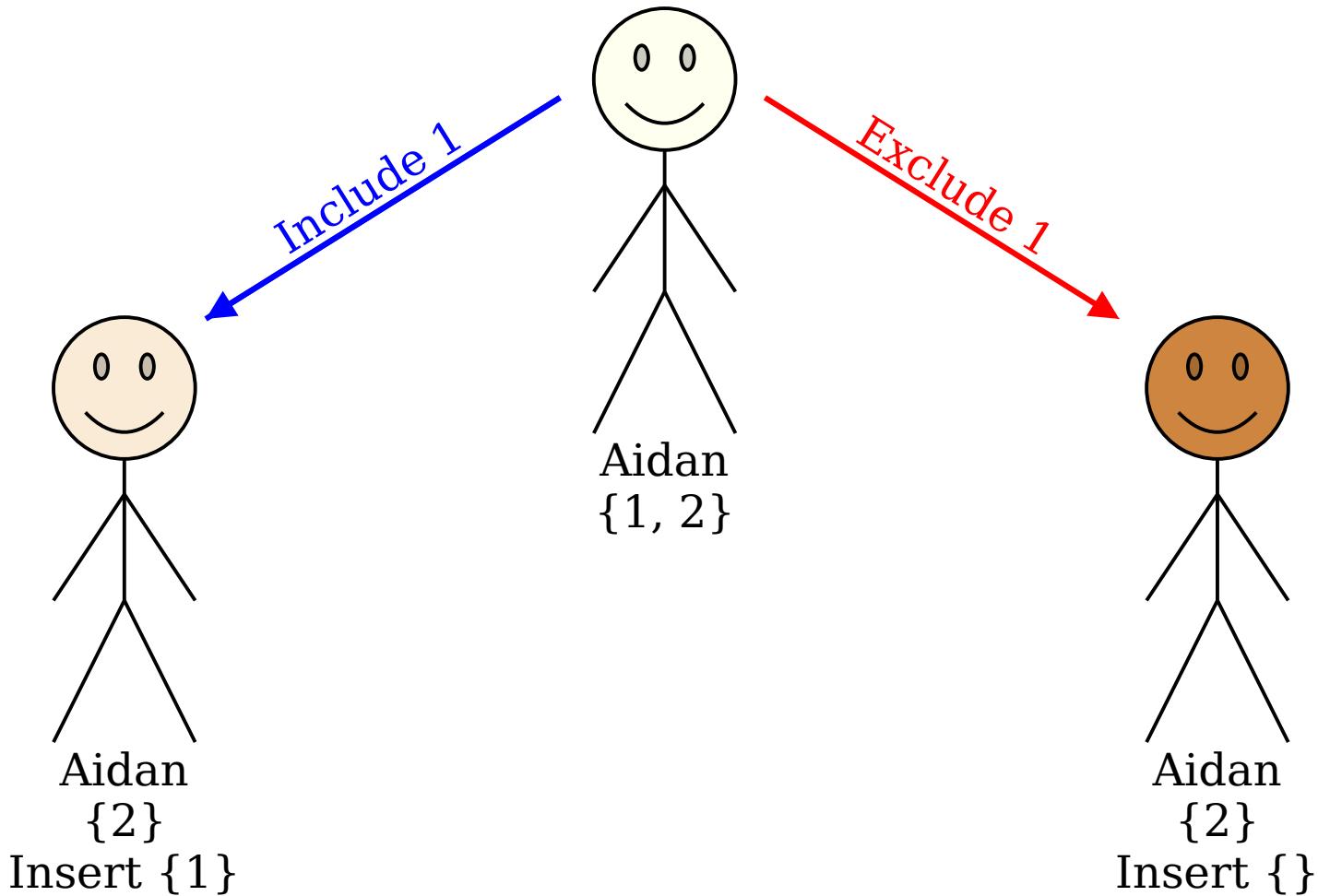
Aidans List Subsets



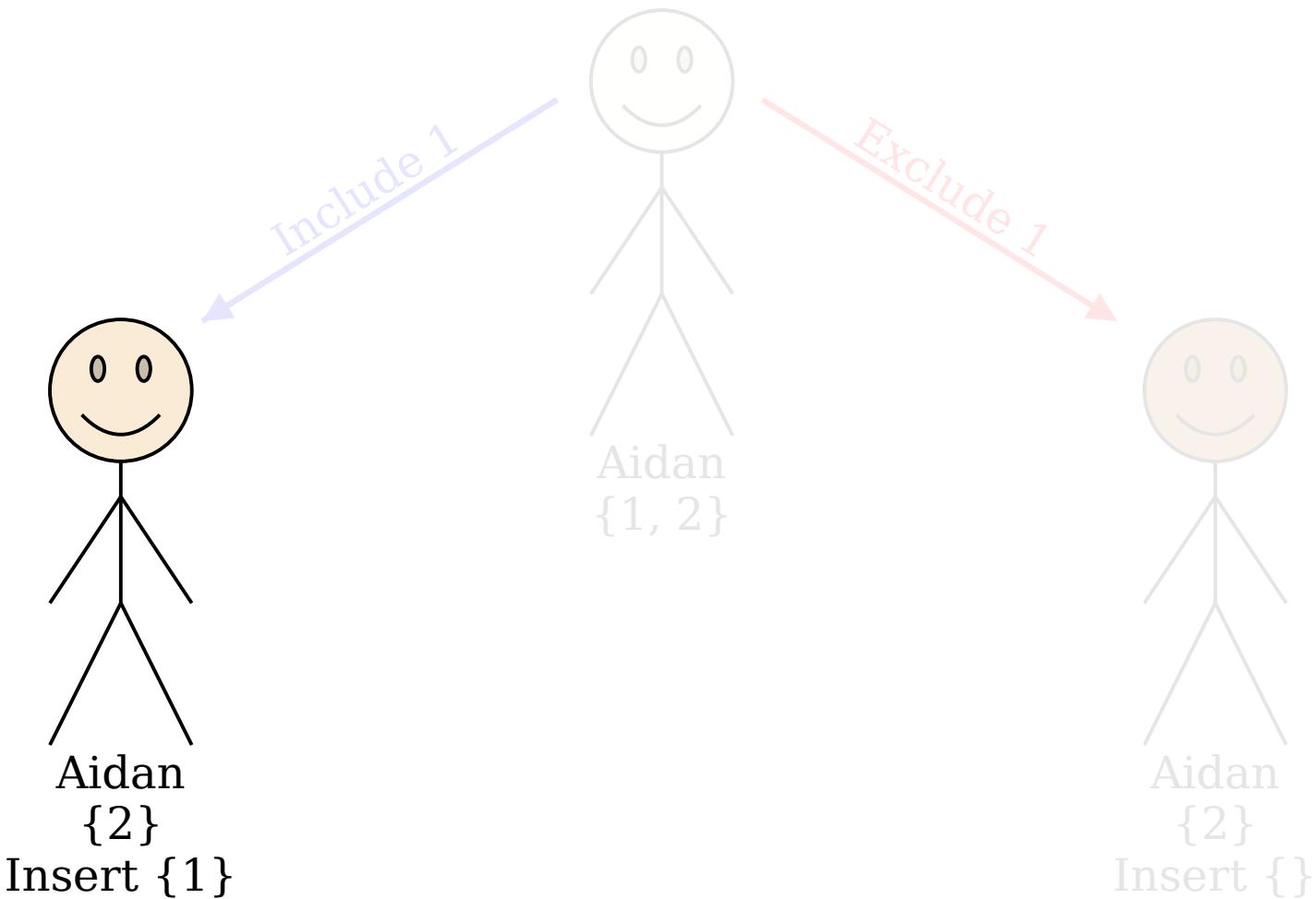
Aidans List Subsets



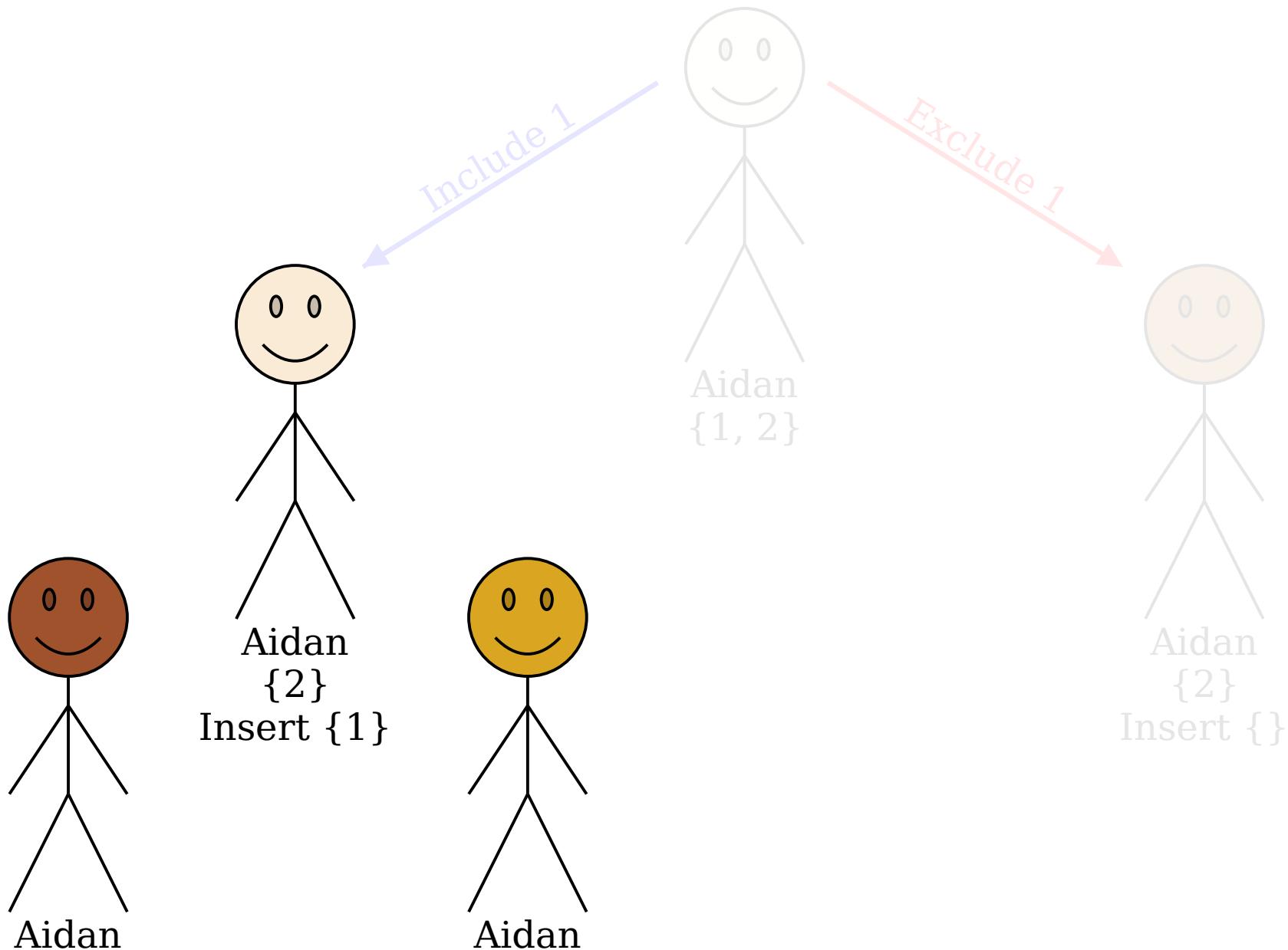
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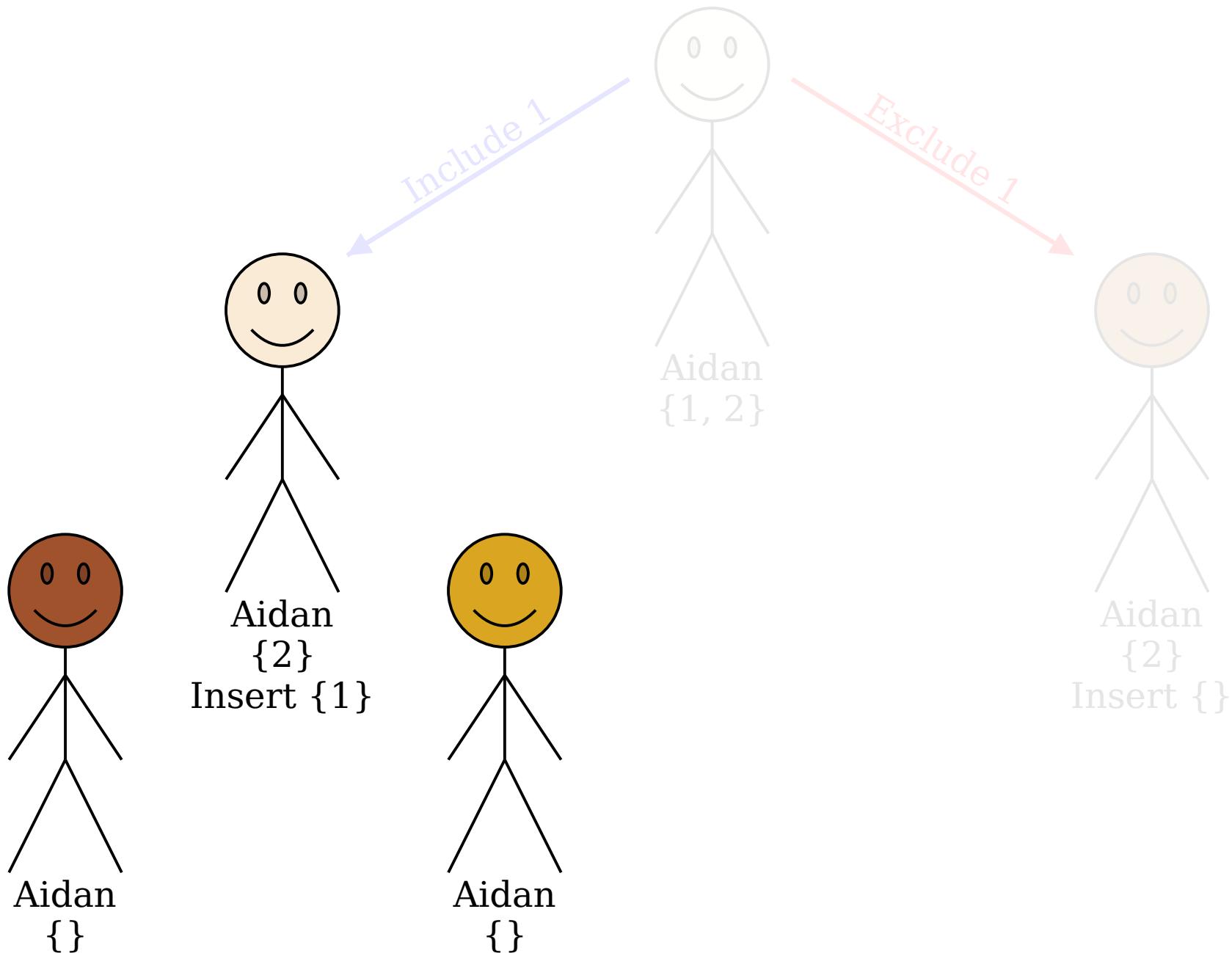
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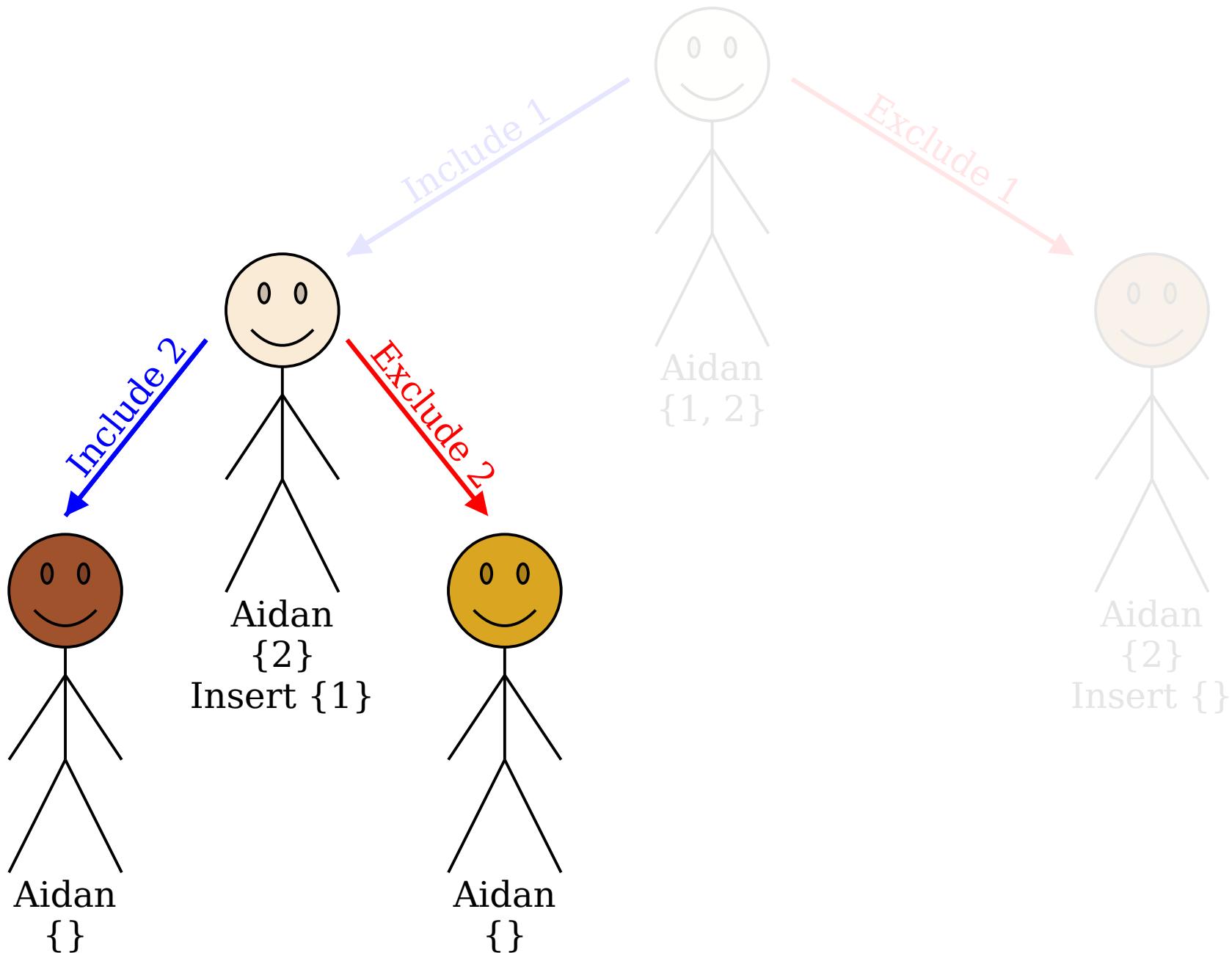
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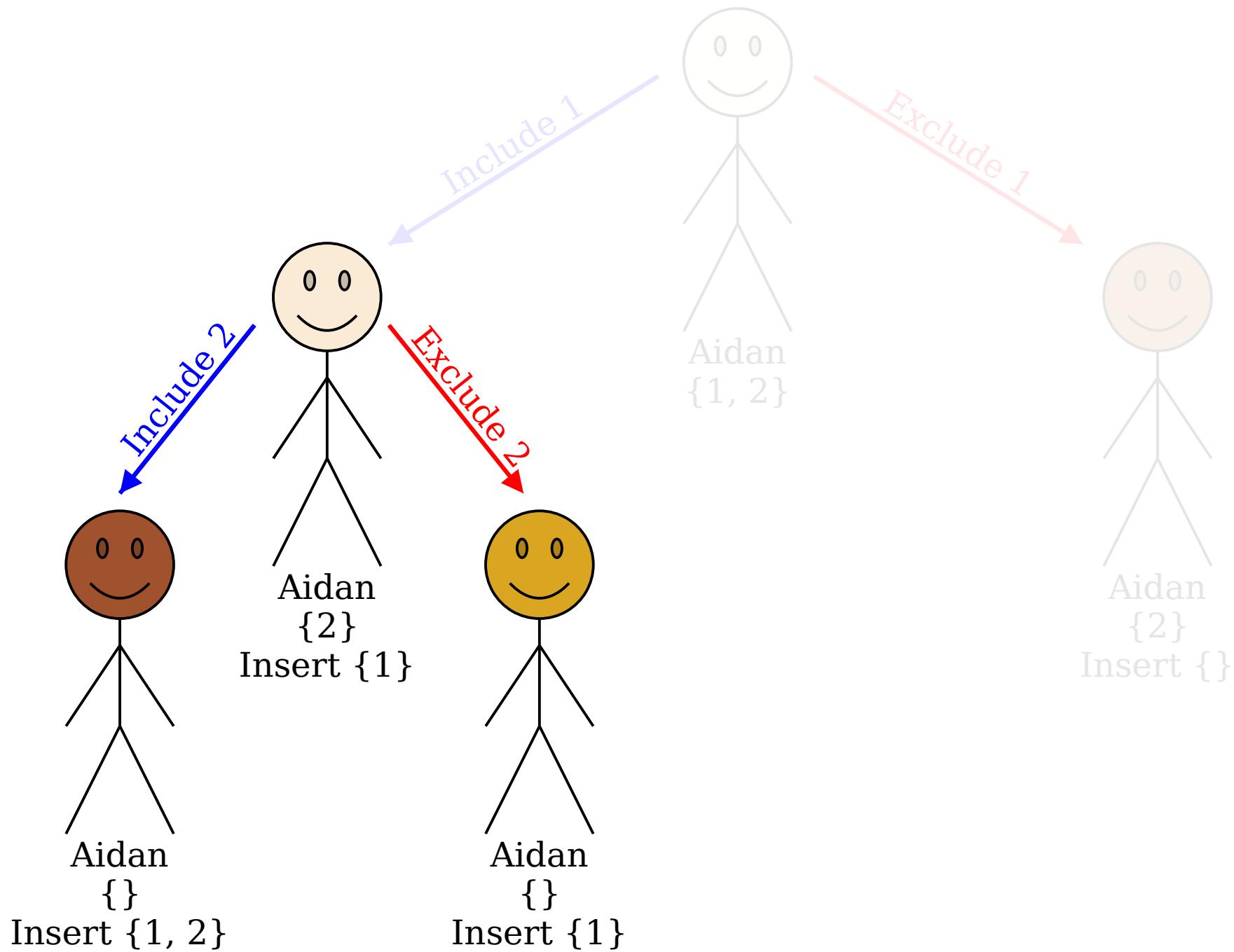
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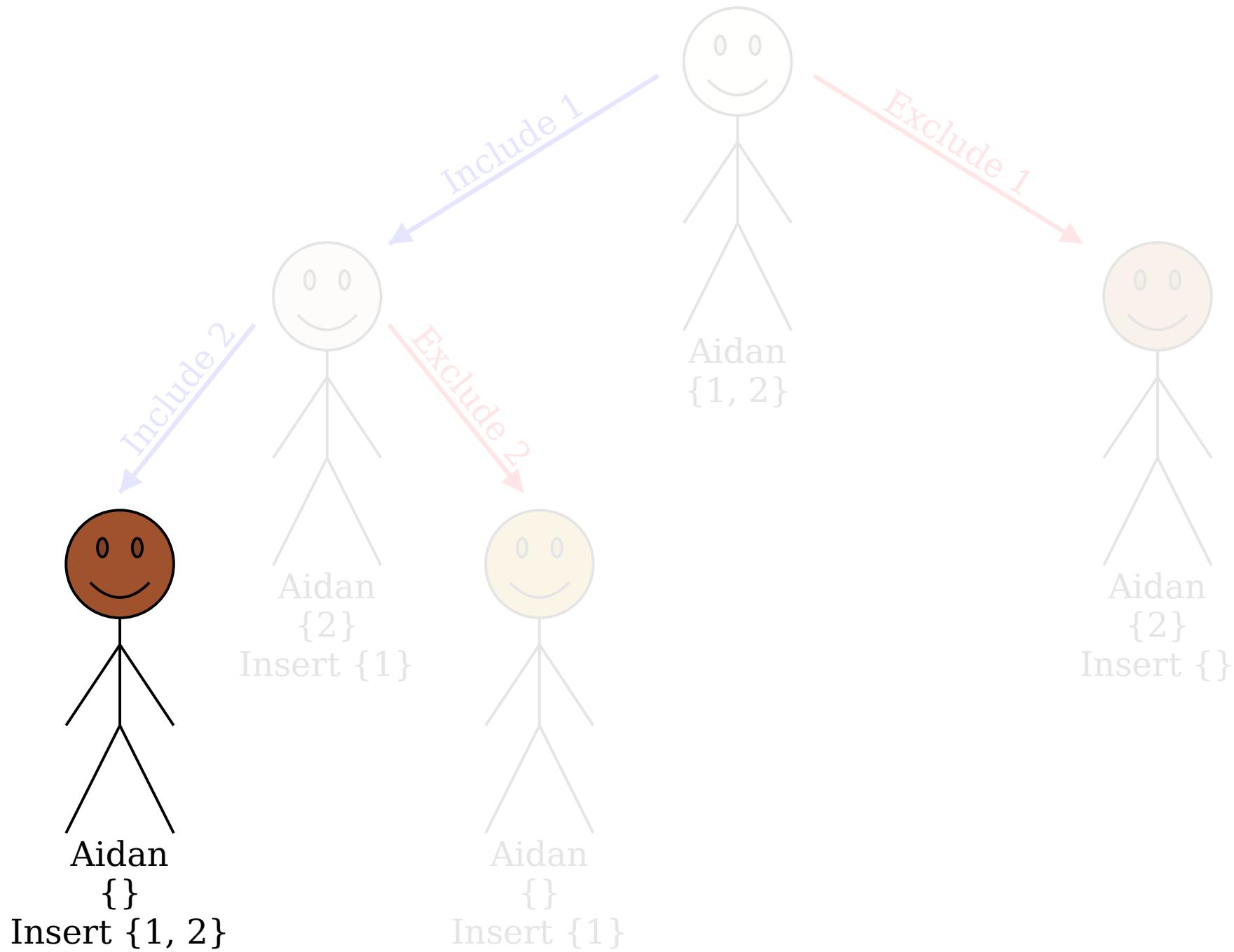
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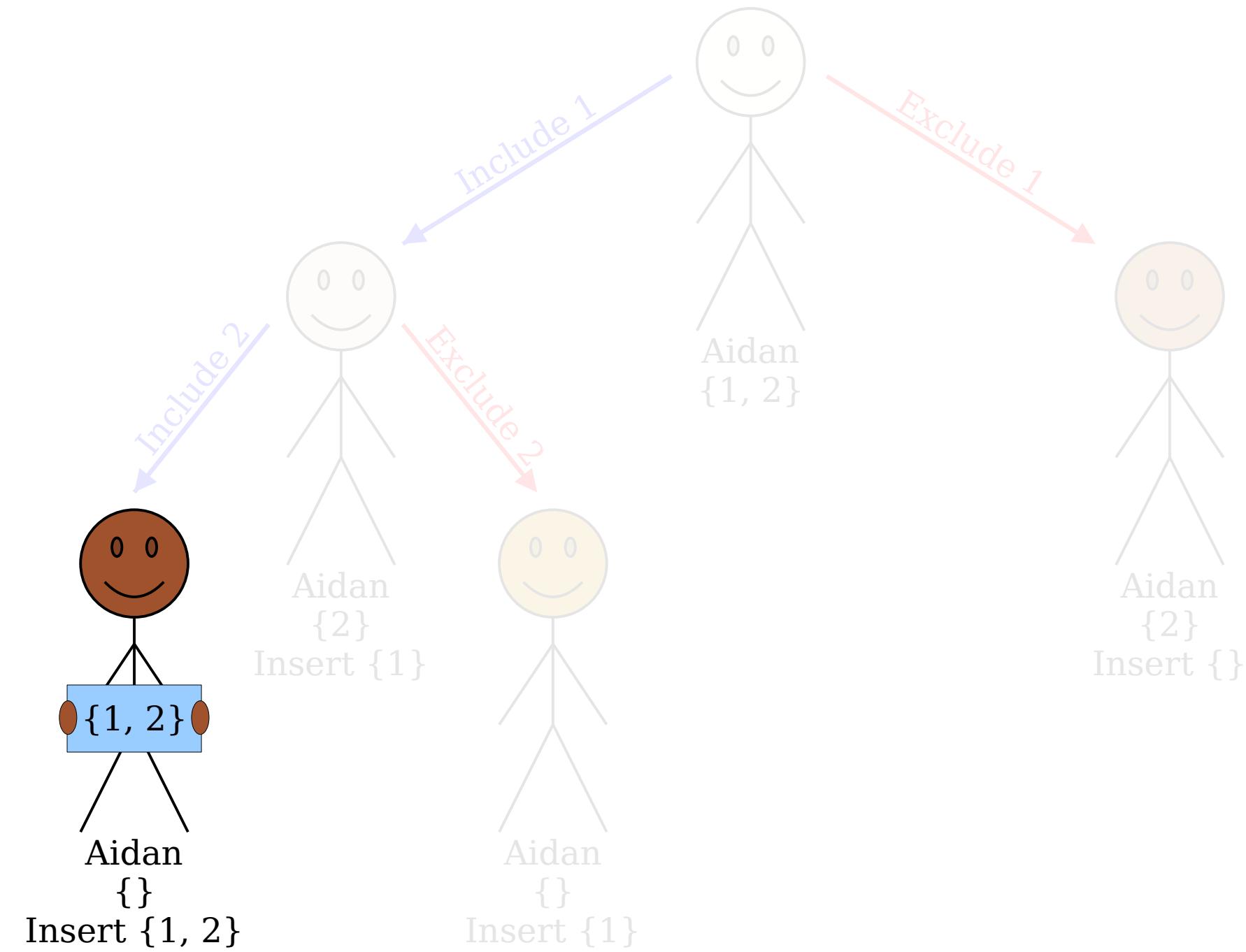
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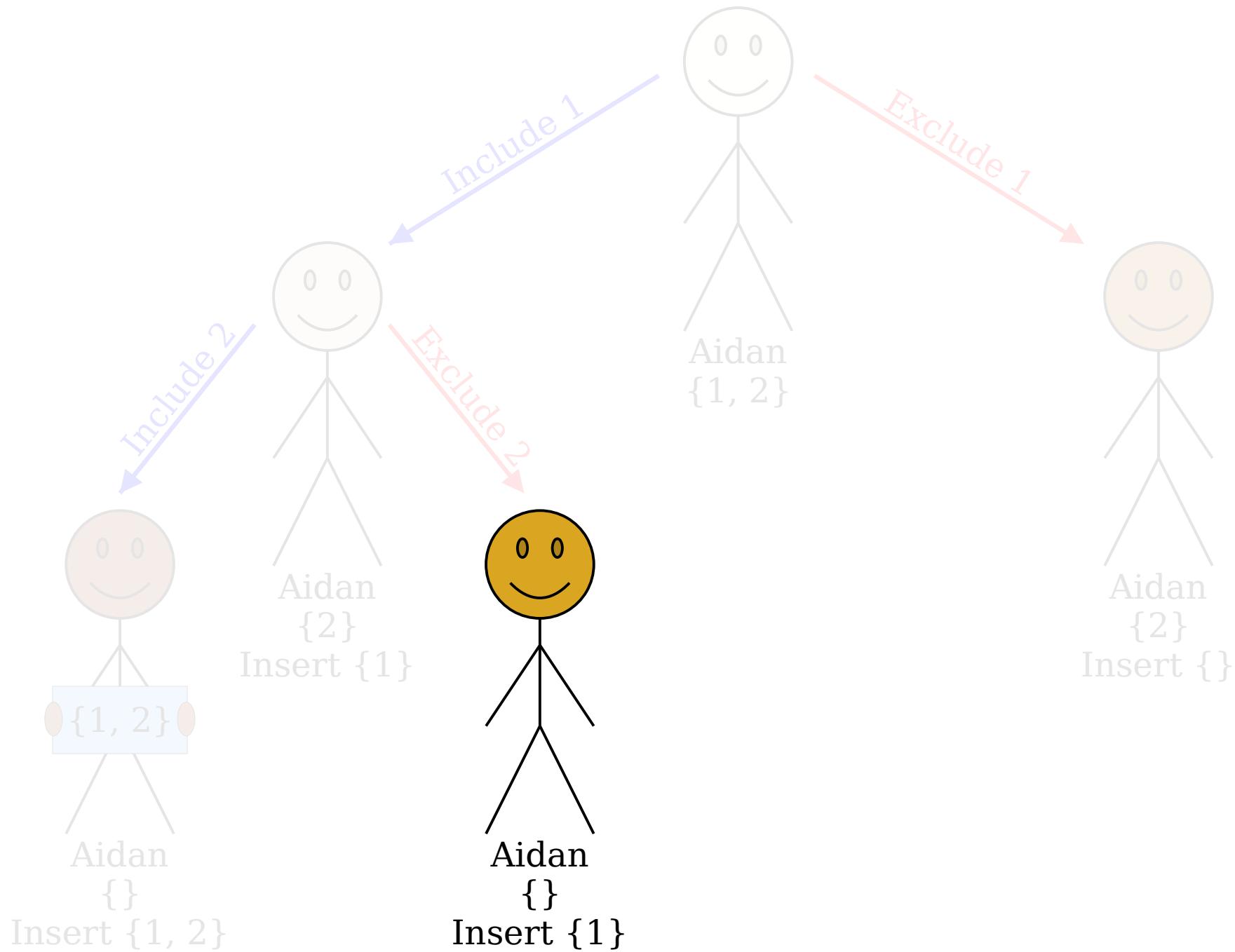
Aidans List Subsets



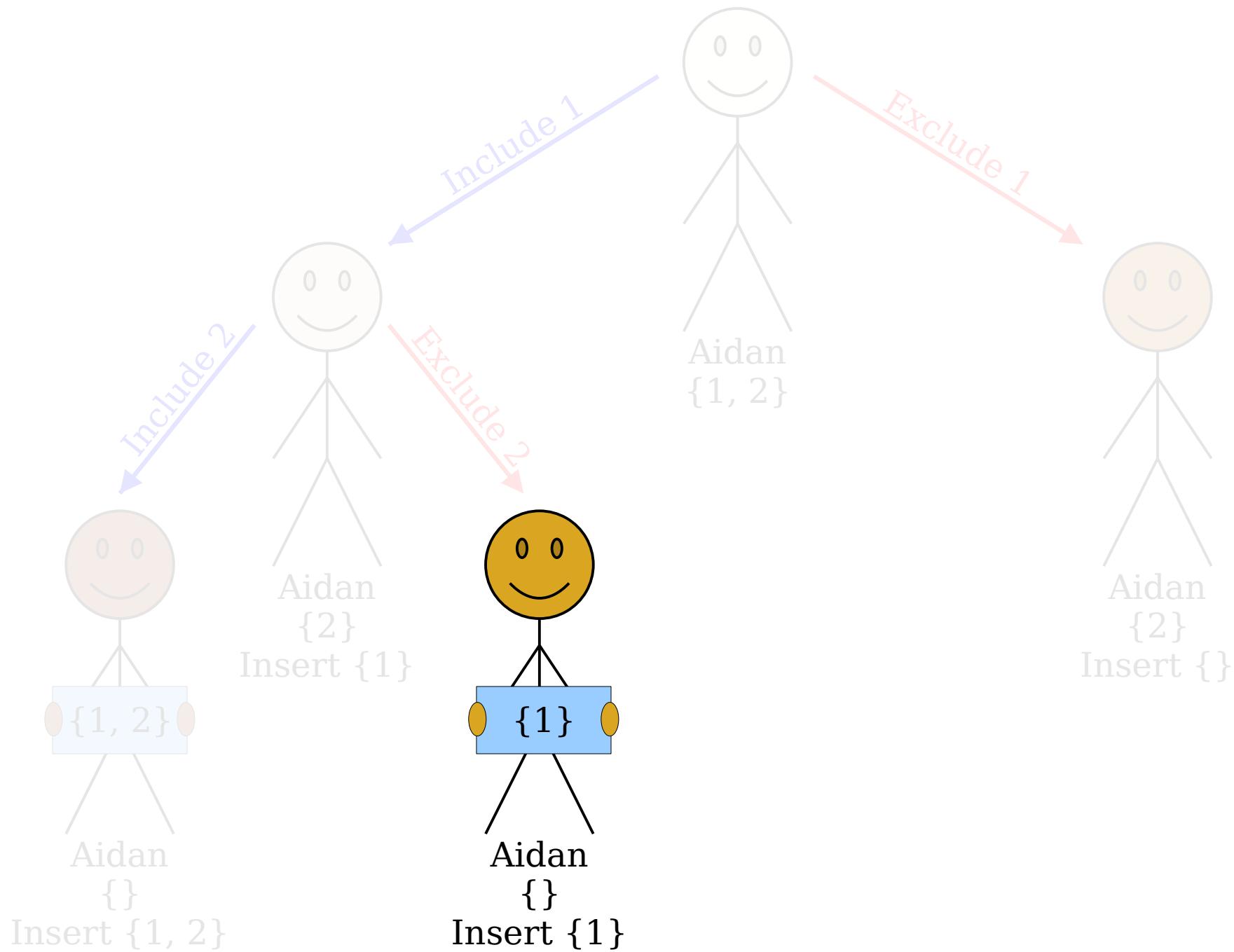
Aidans List Subsets



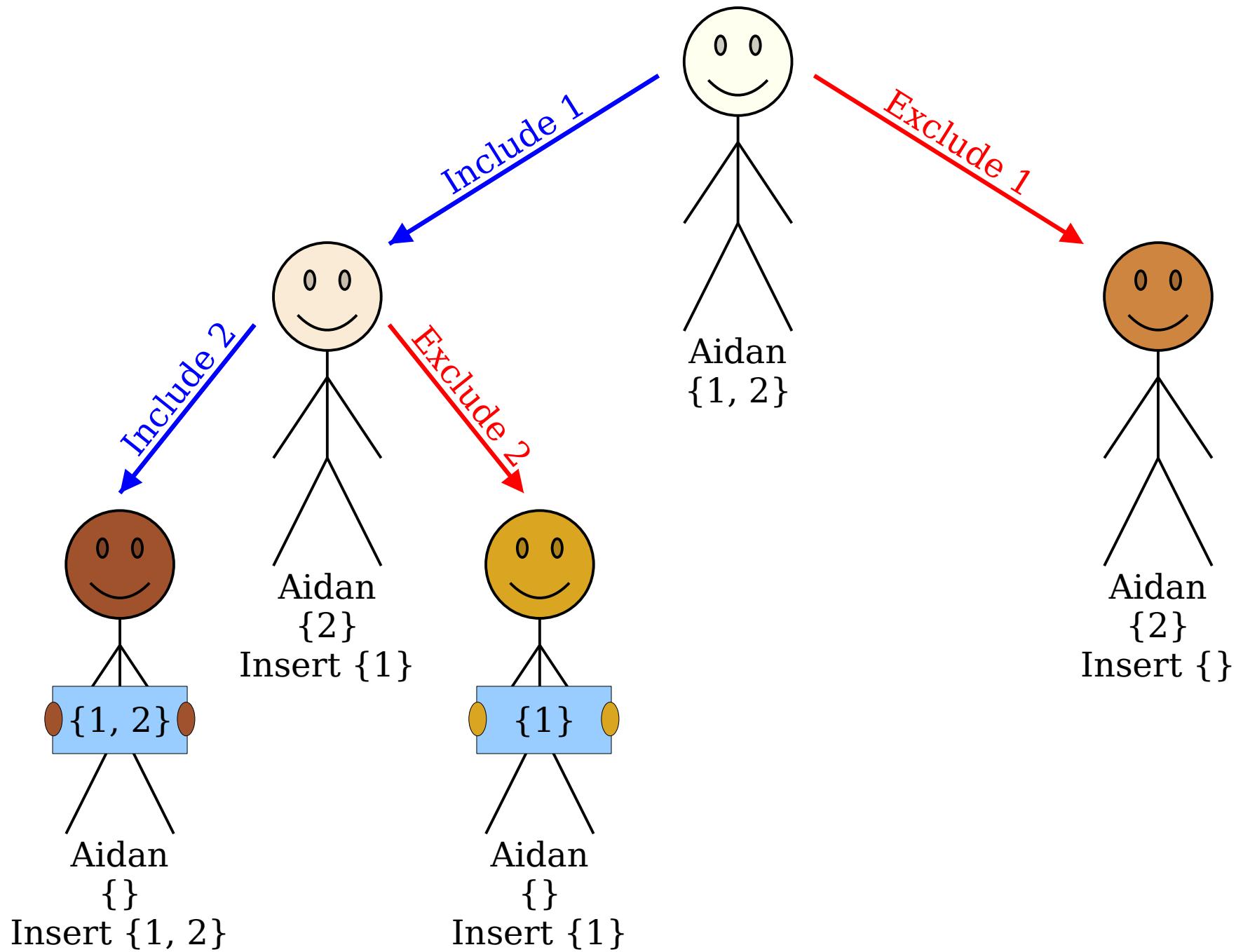
Aidans List Subsets



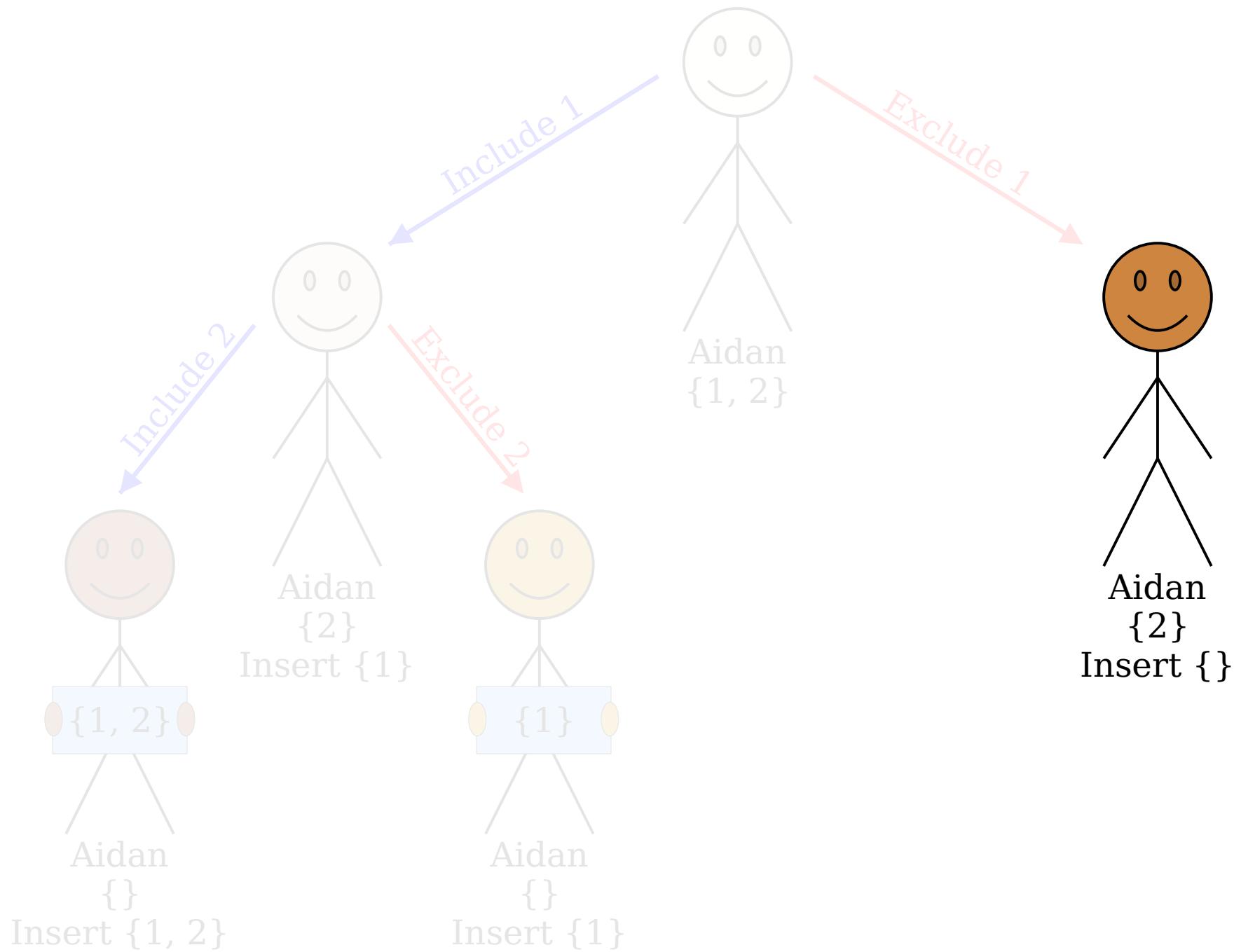
Aidans List Subsets



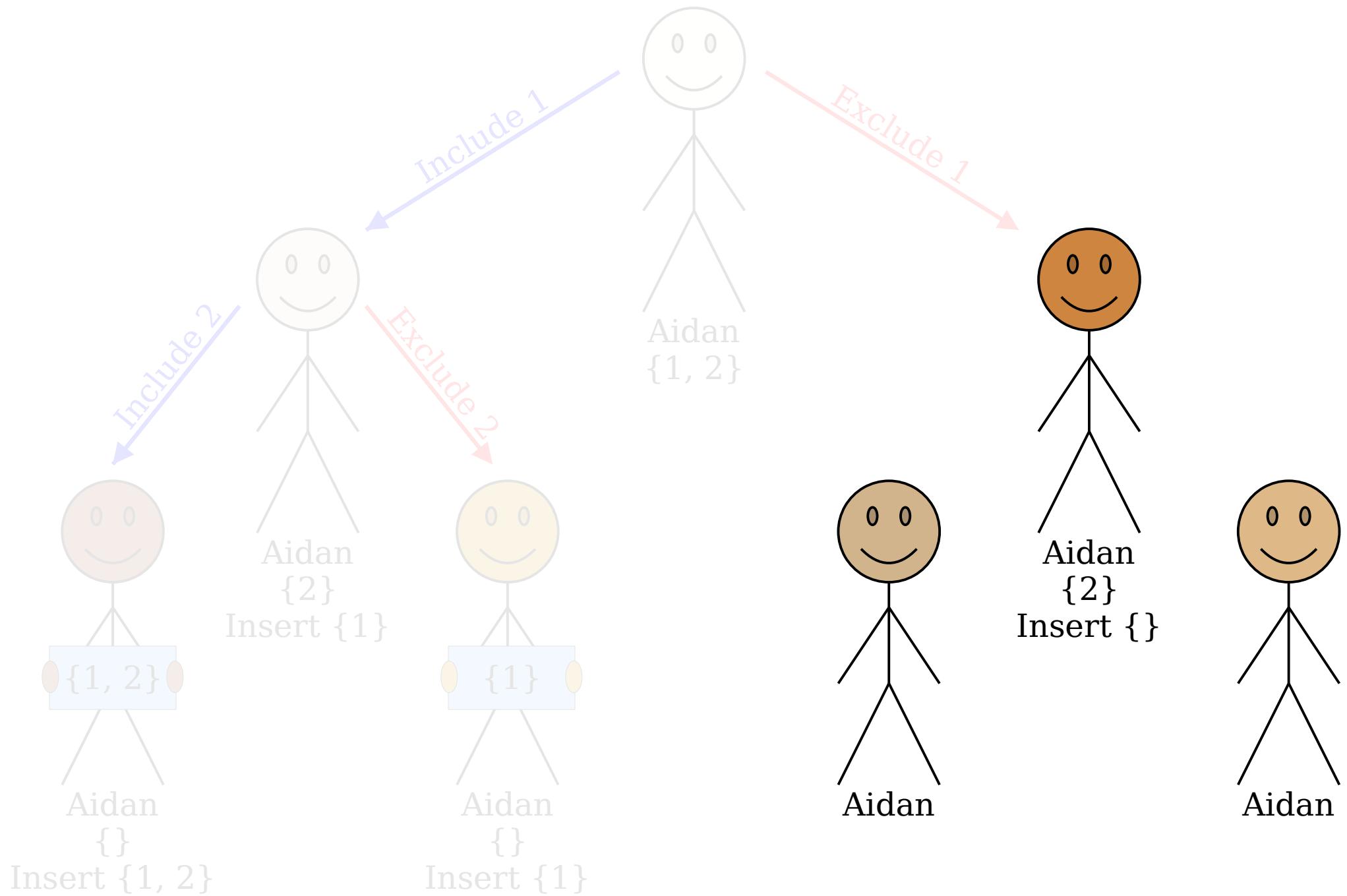
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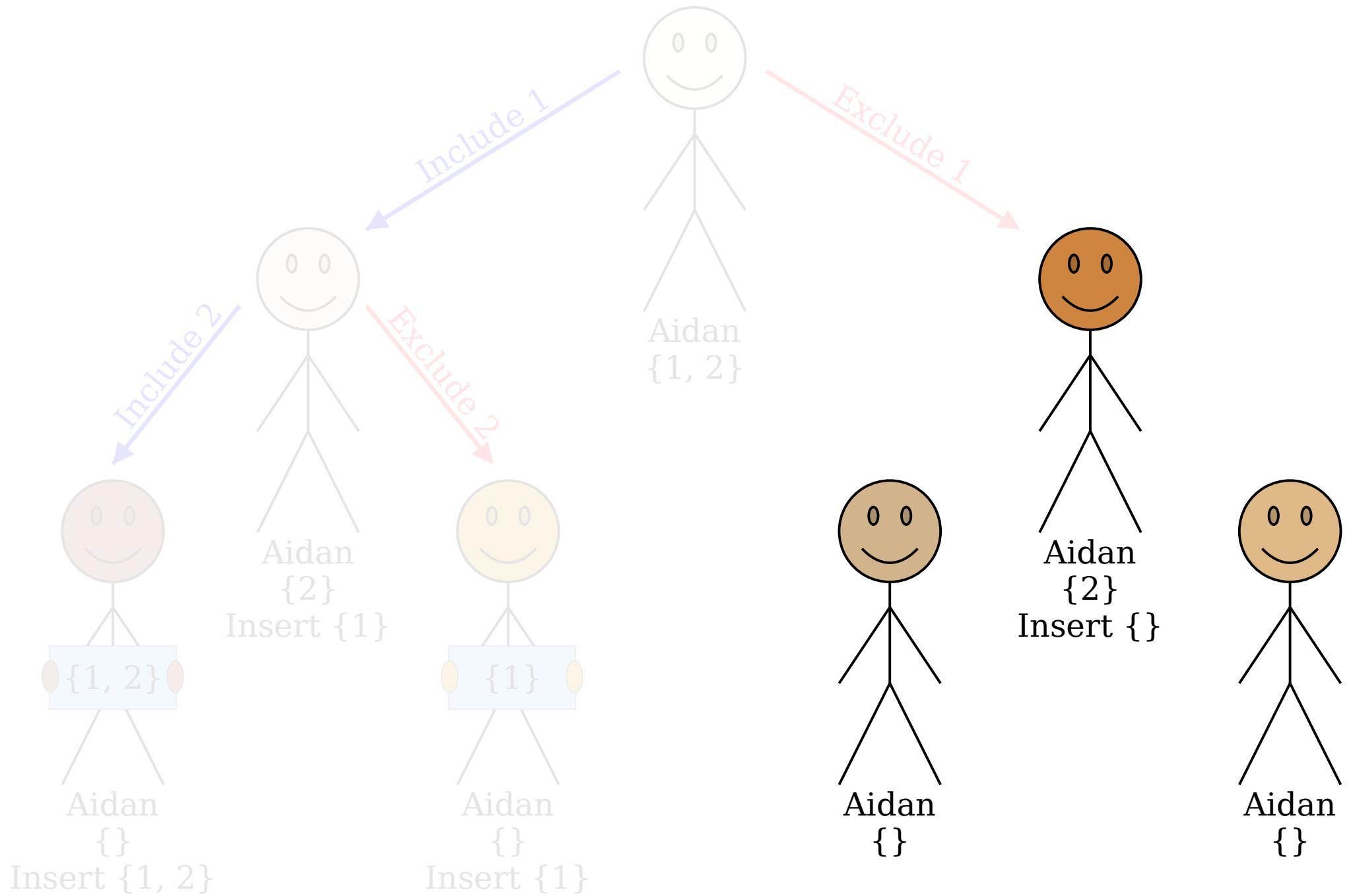
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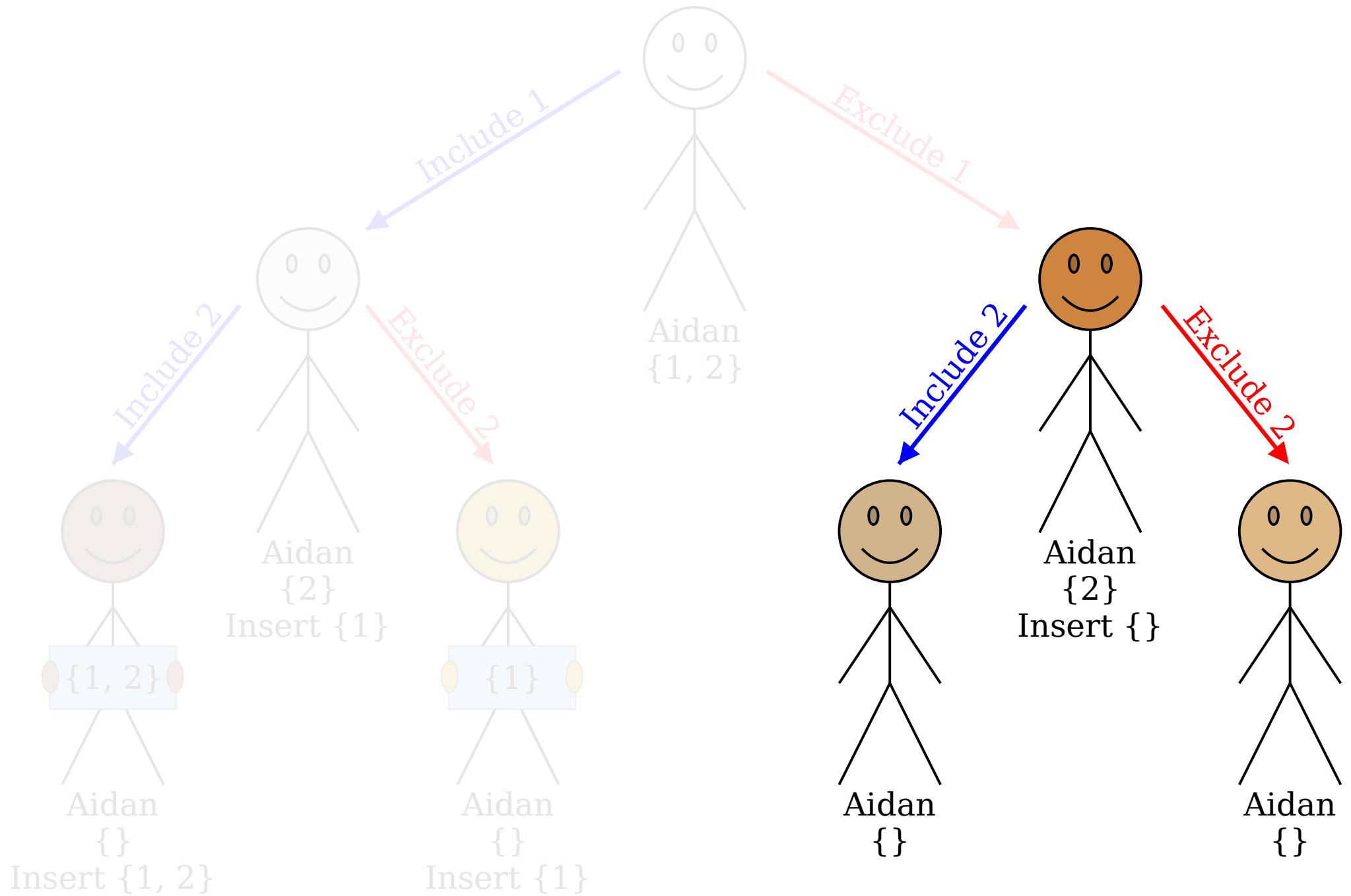
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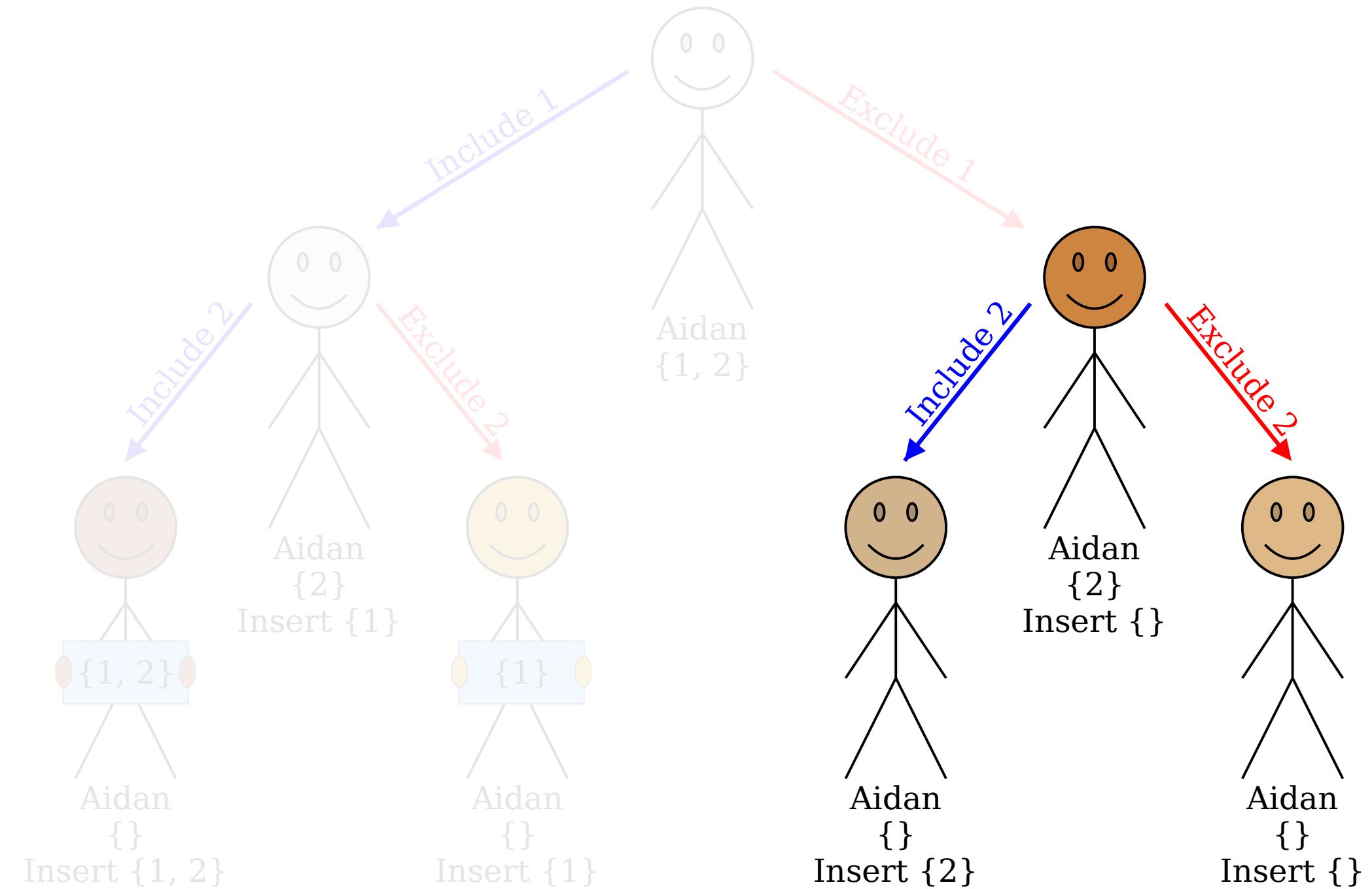
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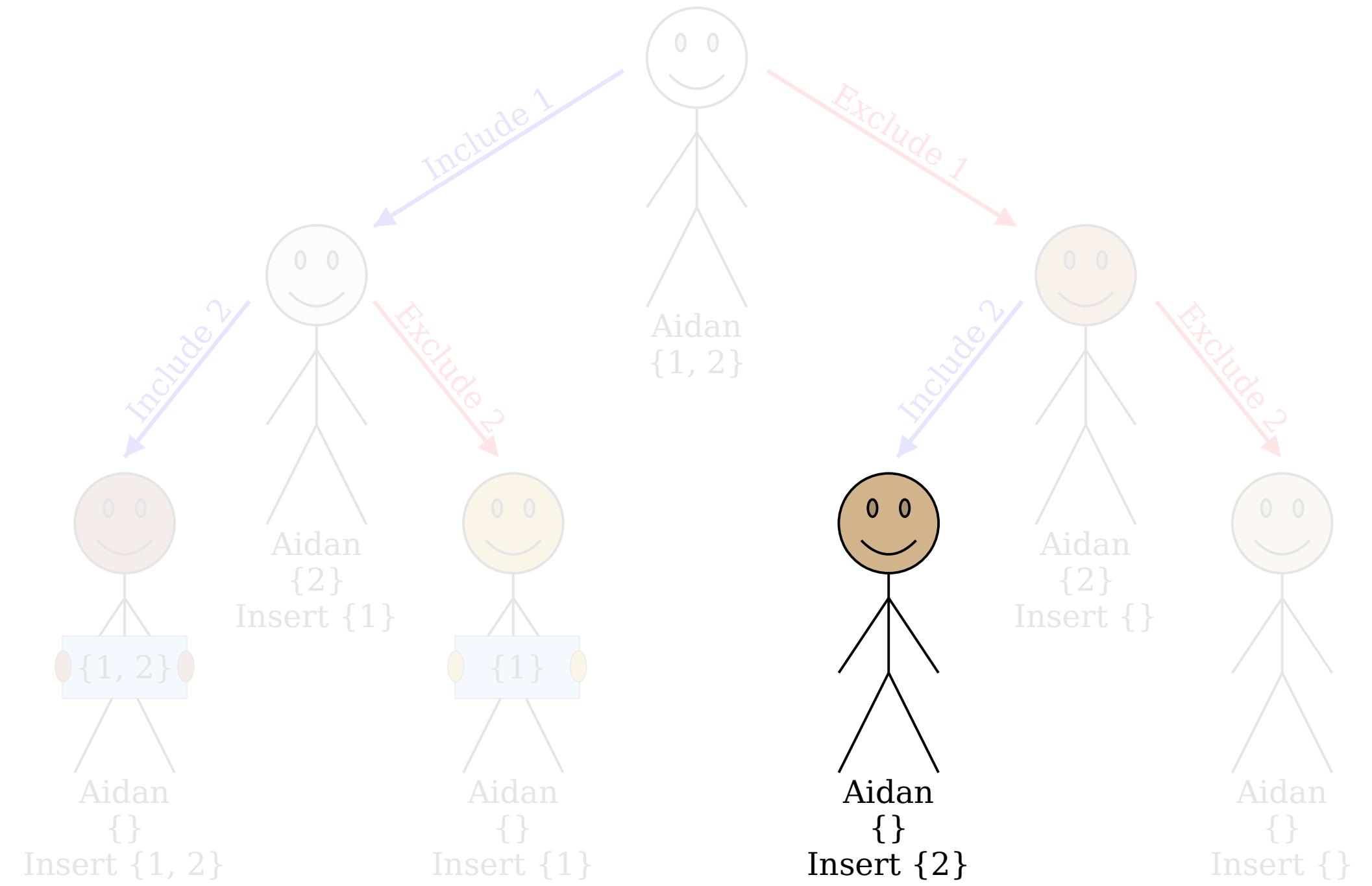
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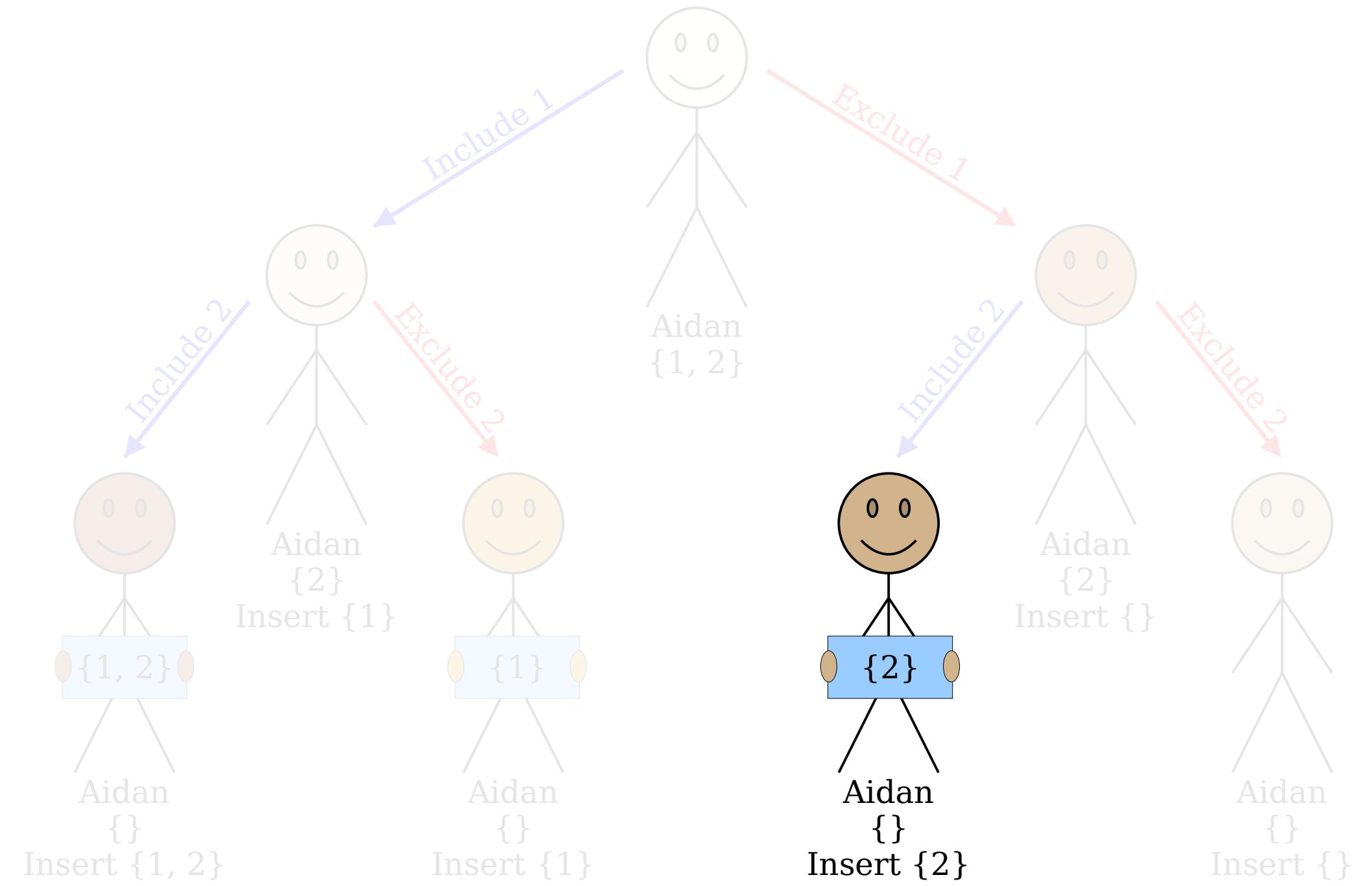
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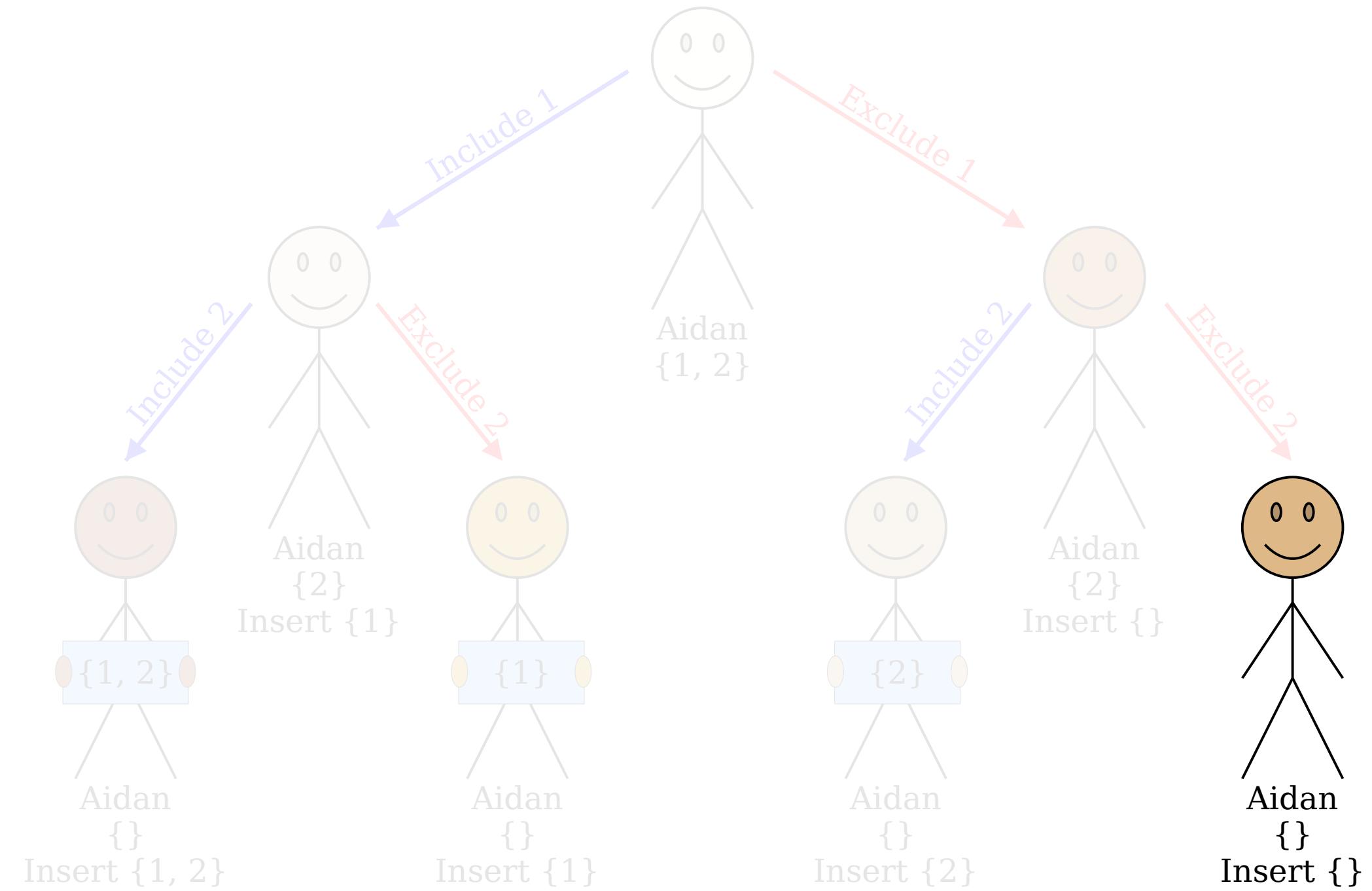
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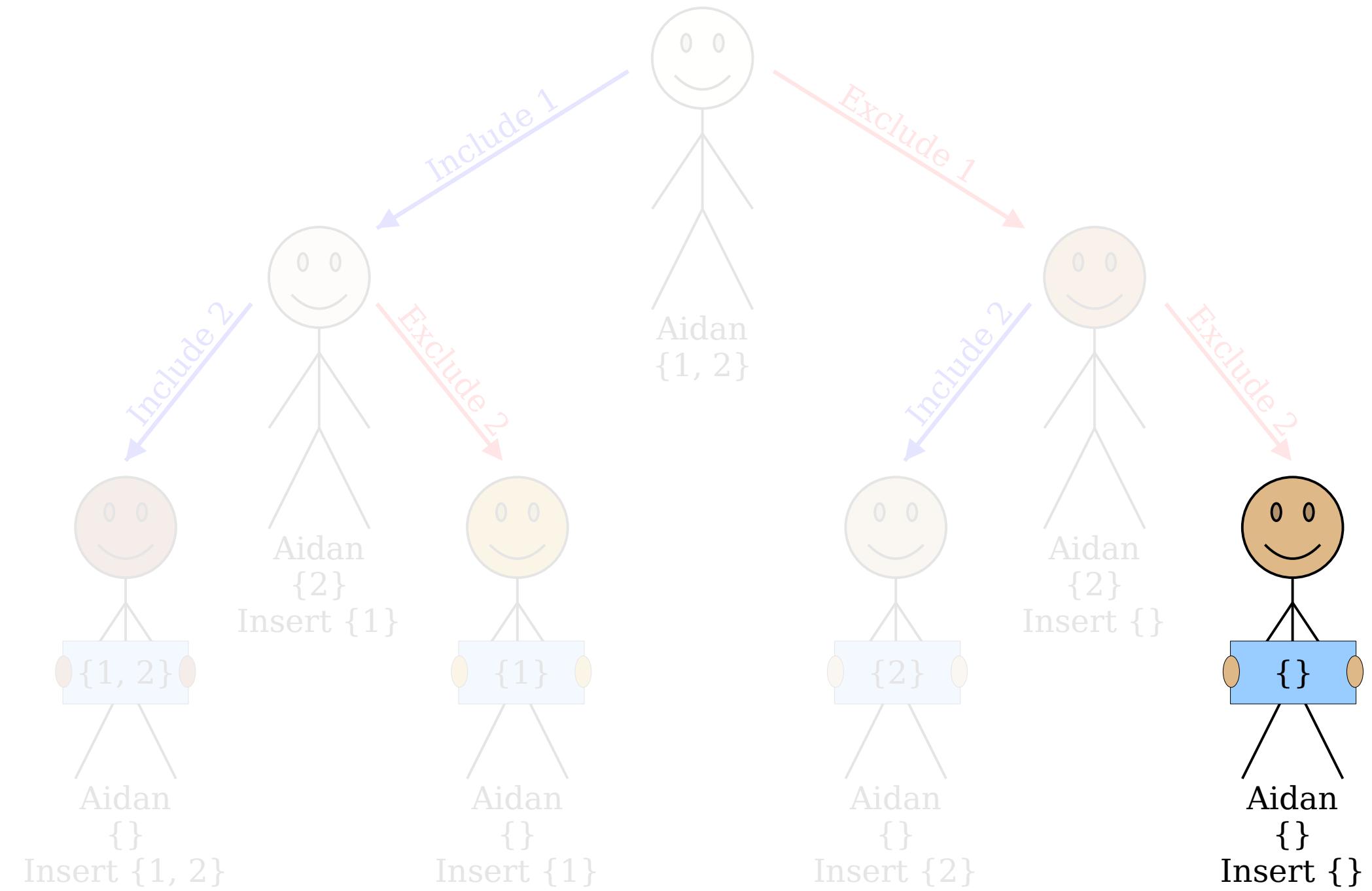
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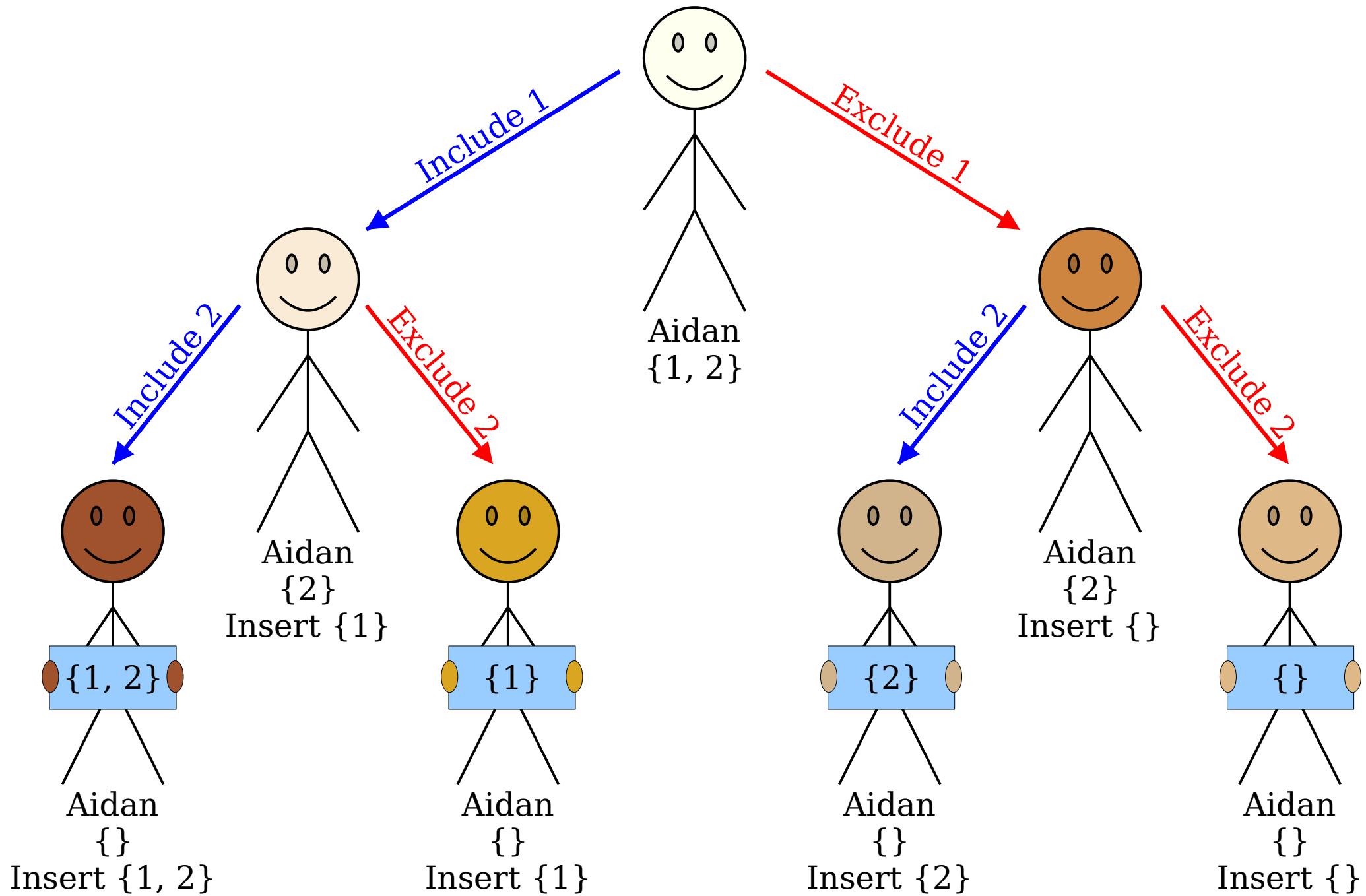
Aidans List Subsets



Aidans List Subsets

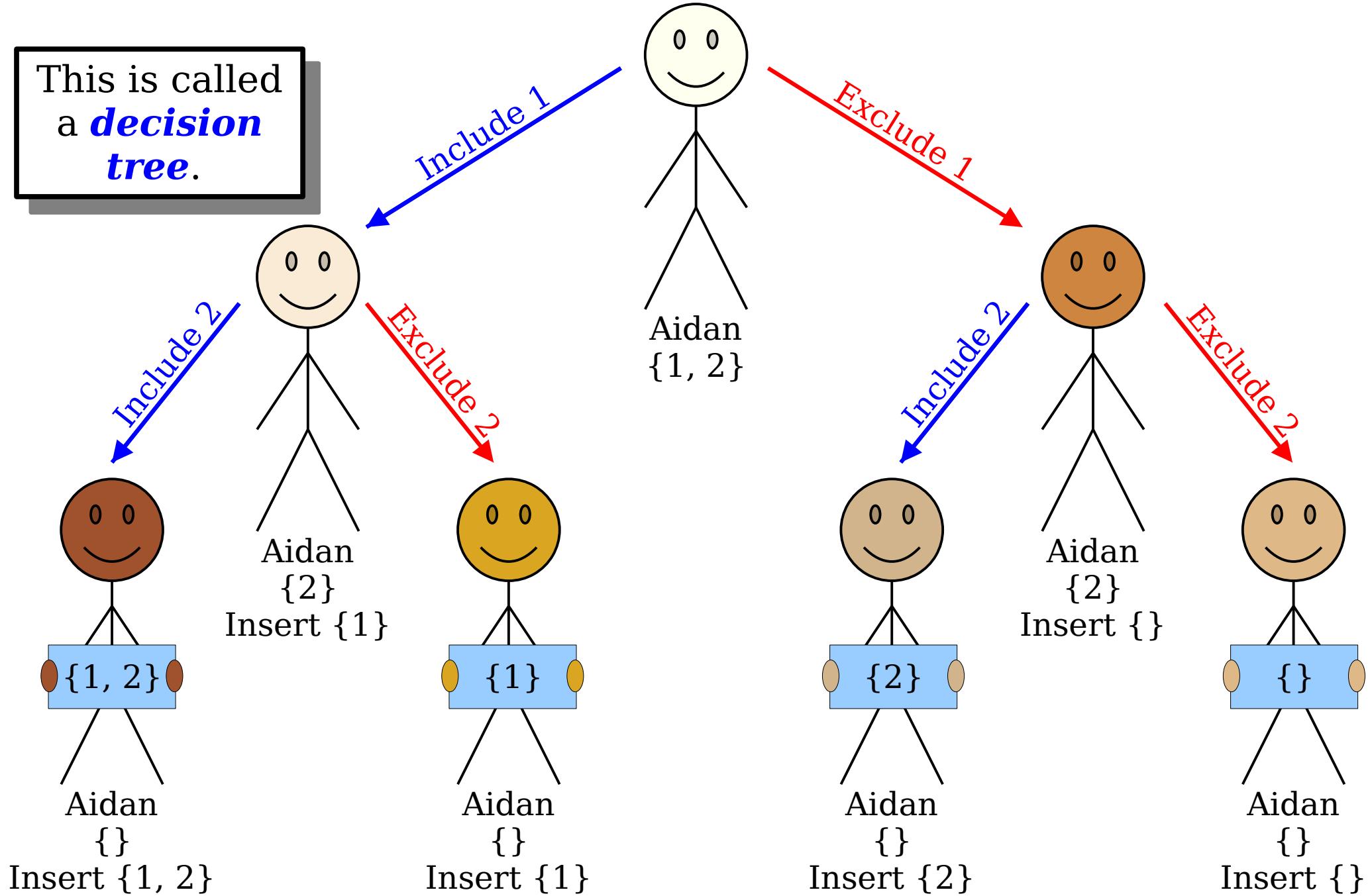


Aidans List Subsets

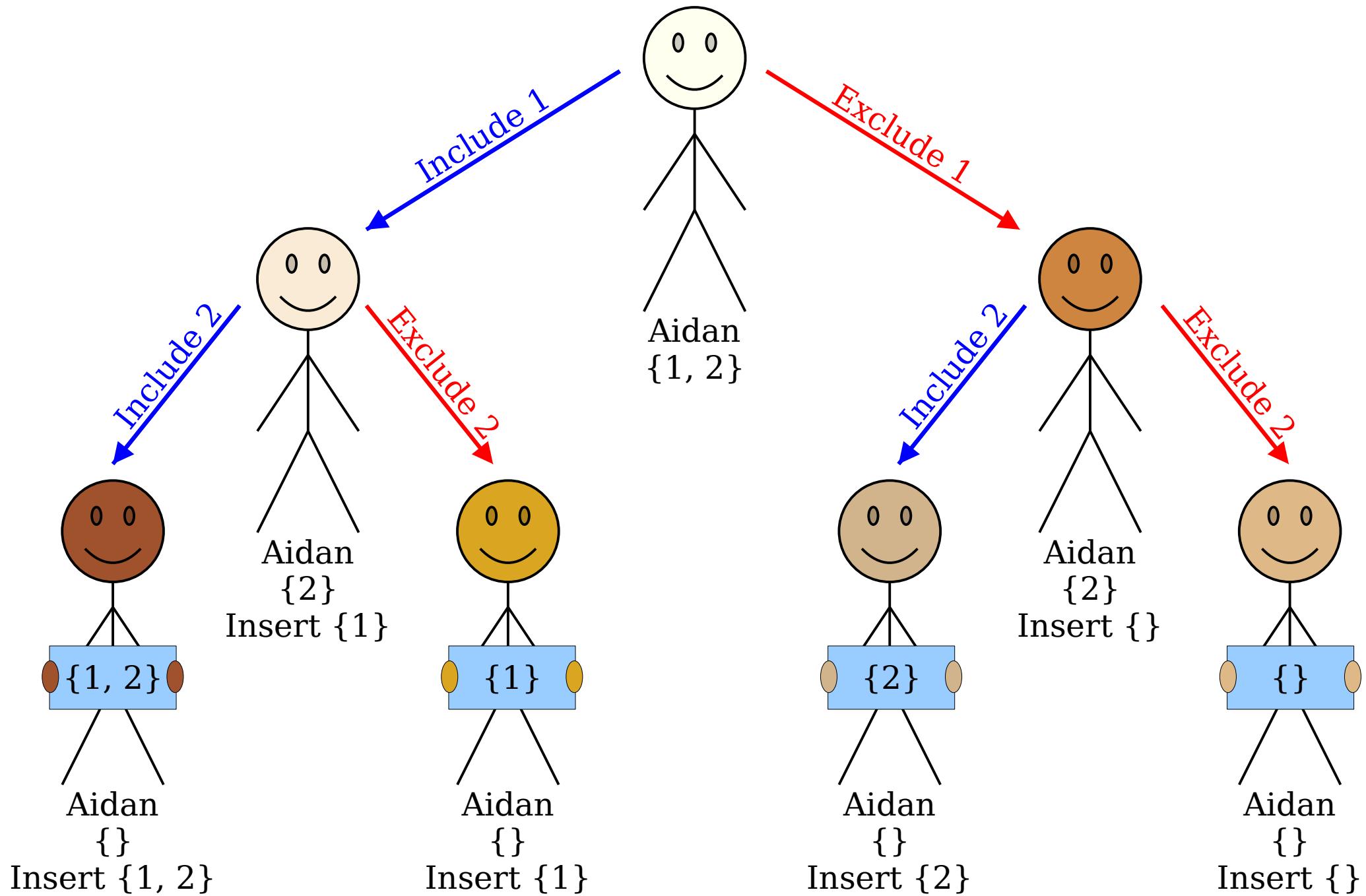


Aidans List Subsets

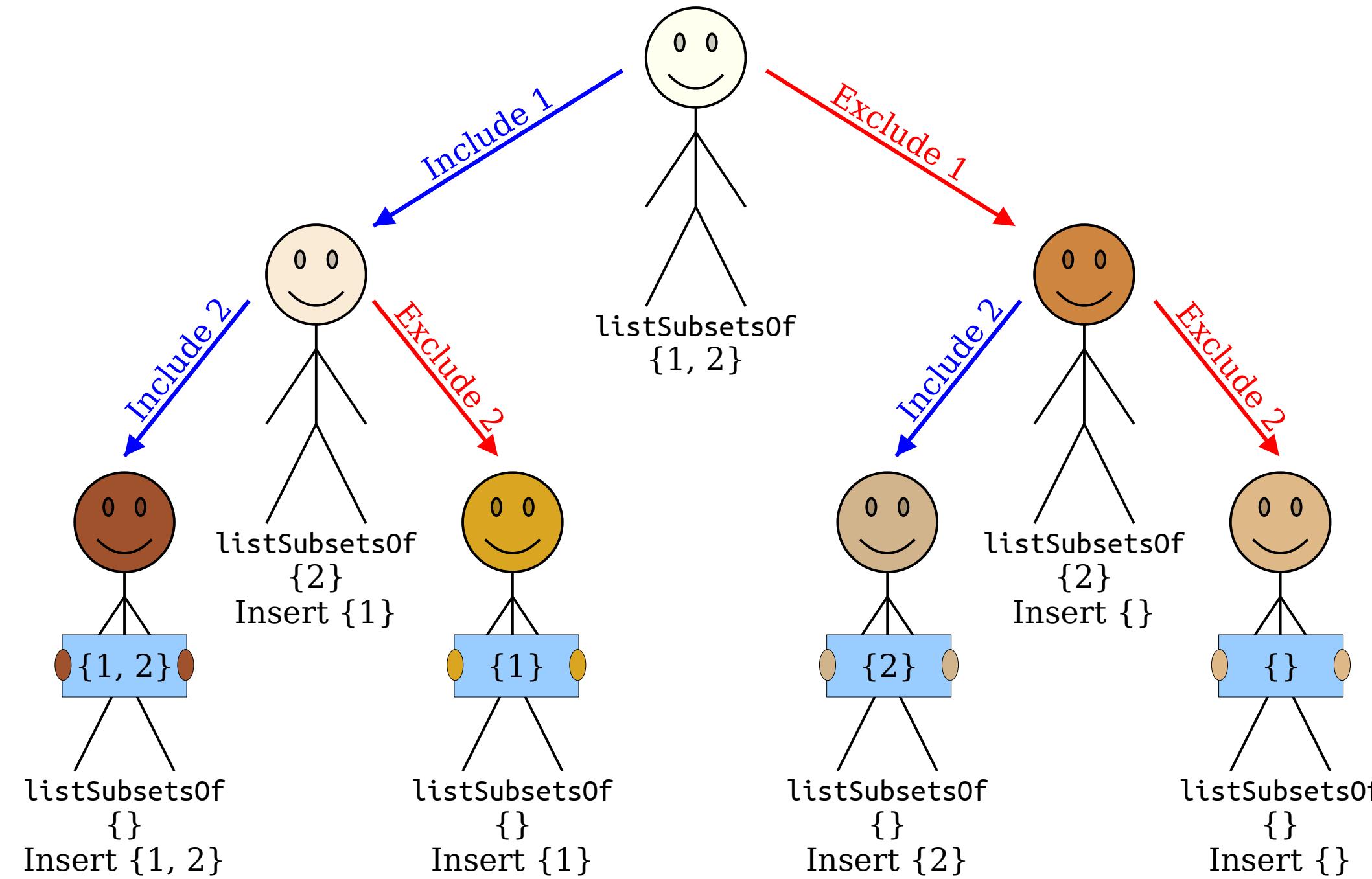
This is called
a **decision
tree**.



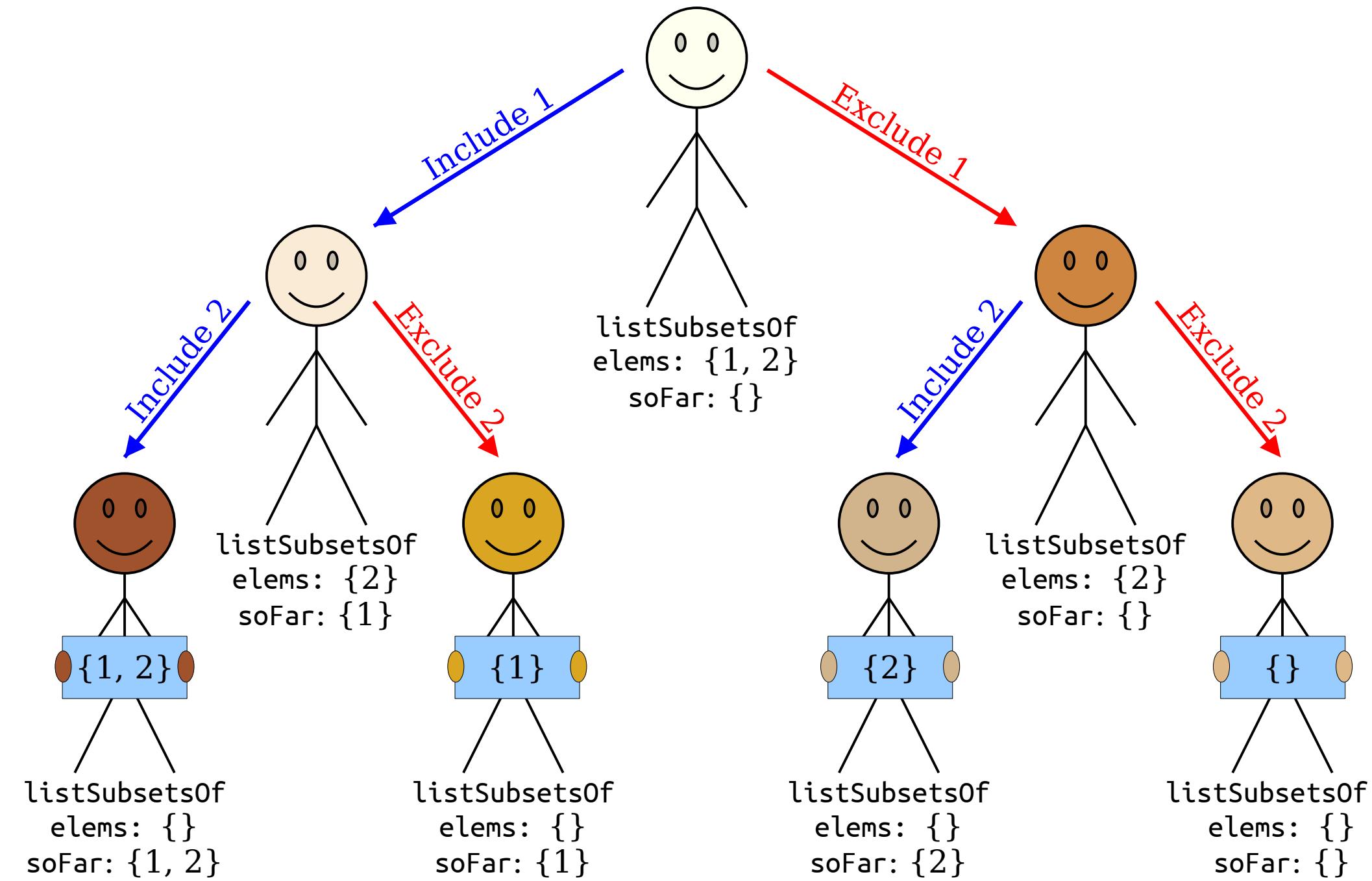
Aidans List Subsets



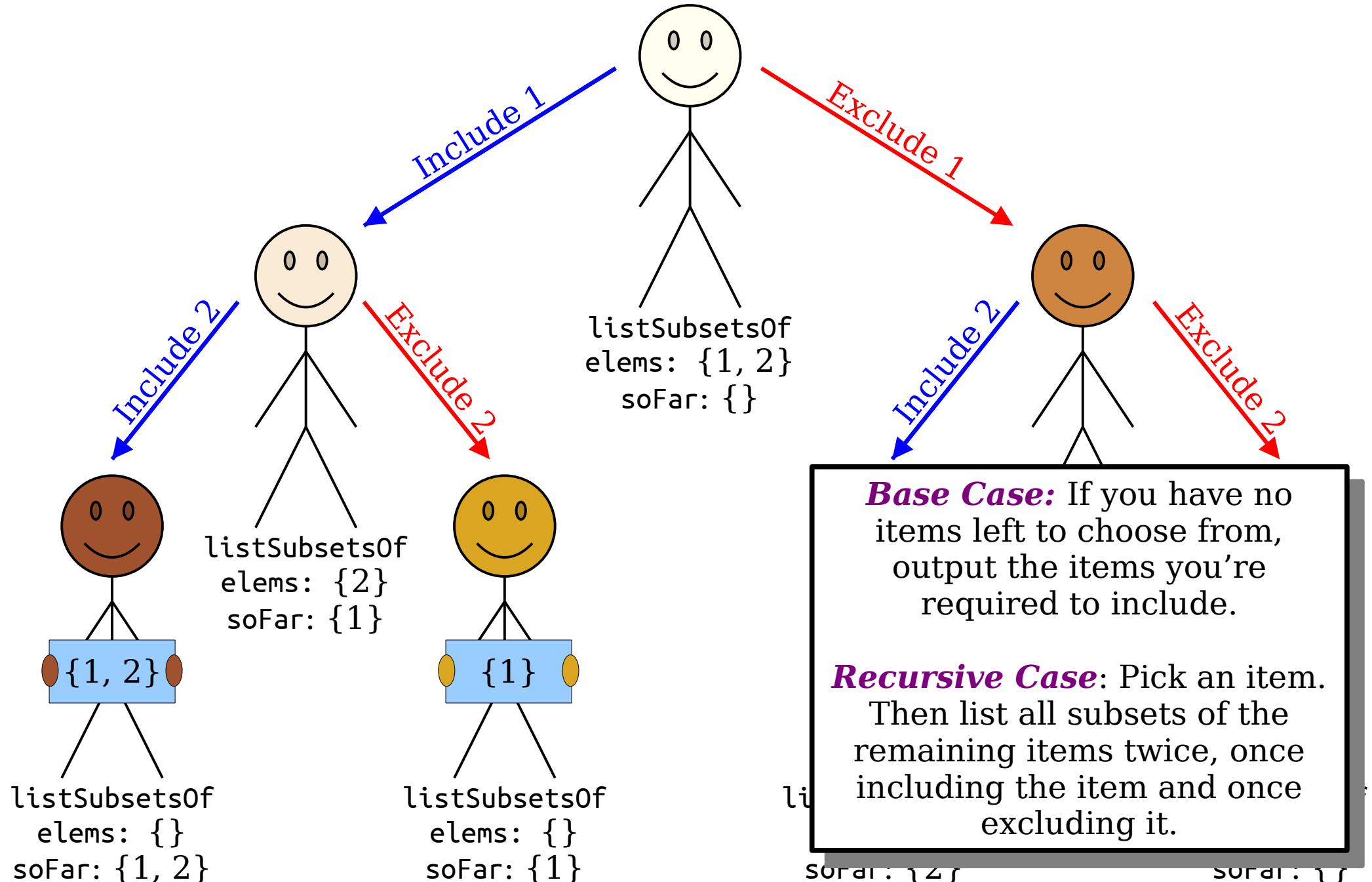
Aidans List Subsets



Aidans List Subsets



Aidans List Subsets



Summary For Today

- Making the ***recursive leap of faith*** and trusting that your recursive calls will perform as expected helps simplify writing recursive code.
- A ***decision tree*** models all the ways you can make choices to arrive at a set of results.

Your Action Items

- ***Read Chapter 8.***
 - There's a lot of great information there about recursive problem-solving, and it's a great resource.
- ***Read the Slide Appendix***
 - There's a trace through how this function works; review this before next lecture.
- ***Finish Assignment 2***
 - If you're following our suggested timetable, at this point you'll have finished Rosetta Stone and will have started working on Rising Tides.
 - Come to LaIR or ask on EdStem if you have any questions!

Next Time

- *Iteration + Recursion*
 - Combining two techniques together.
- *Enumerating Permutations*
 - What order should we perform tasks in?

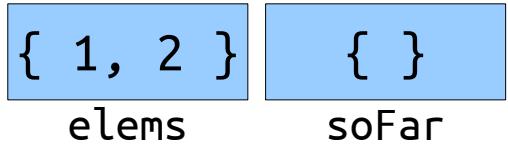
Appendix: Tracing the Recursion

```
int main() {  
    listSubsetsOf({ 1, 2 }, {});  
    return 0;  
}
```

```
int main() {  
    listSubsetsOf({ 1, 2 }, {});  
    return 0;  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        { 1, 2 } { }  
        elems soFar  
    }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        if (elems.isEmpty()) {  
            cout << soFar << endl;  
        } else {  
            int elem = elems.first();  
            Set<int> remaining = elems - elem;  
  
            /* Option 1: Include this element. */  
            listSubsetsOf(remaining, soFar + elem);  
  
            /* Option 2: Exclude this element. */  
            listSubsetsOf(remaining, soFar);  
        }  
    }  
}
```



The diagram illustrates the state of two sets, `elems` and `soFar`, during the execution of the `listSubsetsOf` function. The `elems` set is shown as a blue box containing the elements `{ 1, 2 }`. The `soFar` set is shown as a blue box containing the elements `{ }`.

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        { 1, 2 } { }  
        elems soFar  
    }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        { 1, 2 } { }  
        elems soFar  
    }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        { 1, 2 } { }  
        elems soFar  
    }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
        1 elem  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

```
int main() {
```

```
void listSubsetsOf(const Set<int>& elems,  
                   const Set<int>& soFar) {
```

{ 1, 2 }

{ }

elems

soFar

```
}
```

```
if (elems.isEmpty()) {  
    cout << soFar << endl;
```

```
} else {
```

```
    int elem = elems.first();  
    Set<int> remaining = elems - elem;
```

1

elem

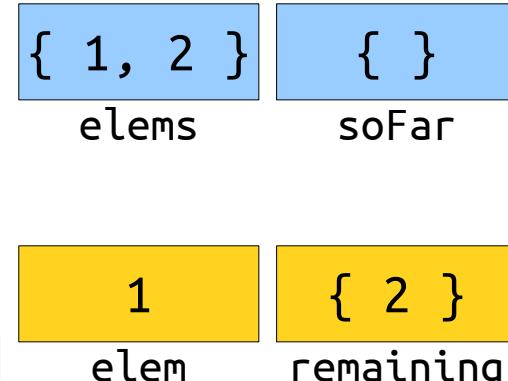
```
    /* Option 1: Include this element. */  
    listSubsetsOf(remaining, soFar + elem);
```

```
    /* Option 2: Exclude this element. */  
    listSubsetsOf(remaining, soFar);
```

```
}
```

```
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        if (elems.isEmpty()) {  
            cout << soFar << endl;  
        } else {  
            int elem = elems.first();  
            Set<int> remaining = elems - elem;  
            /* Option 1: Include this element. */  
            listSubsetsOf(remaining, soFar + elem);  
            /* Option 2: Exclude this element. */  
            listSubsetsOf(remaining, soFar);  
        }  
    }  
}
```



The diagram illustrates the state of variables at a specific point in the recursion. It shows two sets: 'elems' containing the elements {1, 2} and 'soFar' containing the empty set {}, both represented in blue boxes. Below these, the variable 'elem' is shown as the value 1 in a yellow box, and the variable 'remaining' is shown as the set {2} in another yellow box.

```
int main() {
```

```
    void listSubsetsOf(const Set<int>& elems,
                        const Set<int>& soFar) {
        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
            Set<int> remaining = elems - elem;
```

1 { 2 }
elem remaining

```
                /* Option 1: Include this element. */
                listSubsetsOf(remaining, soFar + elem);

                /* Option 2: Exclude this element. */
                listSubsetsOf(remaining, soFar);
    }
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 1, 2 } { } }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) { { 2 } { 1 } }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 1, 2 } { } }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) { { 2 } { 1 } }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 1, 2 } { } }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) { { 2 } { 1 } }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 1, 2 } { } }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) { { 2 } { 1 } }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 1, 2 } { }  
    }  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 2 } { 1 }  
        }  
        if (elems.isEmpty()) {  
            cout << soFar << endl;  
        } else {  
            int elem = elems.first(); { 2 }  
            Set<int> remaining = elems - elem; elem  
            /* Option 1: Include this element. */  
            listSubsetsOf(remaining, soFar + elem);  
            /* Option 2: Exclude this element. */  
            listSubsetsOf(remaining, soFar);  
        }  
    }  
}
```

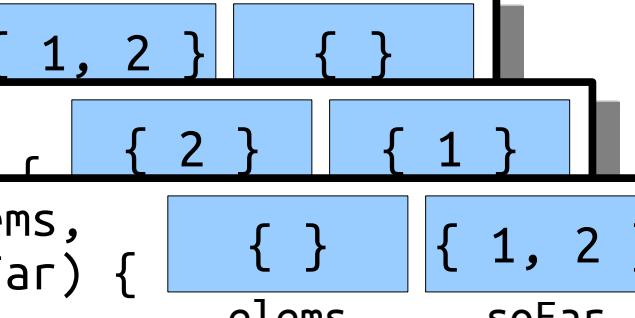
```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar);  
    { 1, 2 } { }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                   const Set<int>& soFar) {  
    { 2 } { 1 }  
    elems soFar  
}  
}  
  
if (elems.isEmpty()) {  
    cout << soFar << endl;  
}  
} else {  
    int elem = elems.first();  
    Set<int> remaining = elems - elem;  
    2 elem  
    /* Option 1: Include this element. */  
    listSubsetsOf(remaining, soFar + elem);  
    /* Option 2: Exclude this element. */  
    listSubsetsOf(remaining, soFar);  
}  
}  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 1, 2 } { }  
    }  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 2 } { 1 }  
        }  
        if (elems.isEmpty()) {  
            cout << soFar << endl;  
        } else {  
            int elem = elems.first(); { }  
            Set<int> remaining = elems - elem; 2 { }  
            /* Option 1: Include this element. */  
            listSubsetsOf(remaining, soFar + elem);  
            /* Option 2: Exclude this element. */  
            listSubsetsOf(remaining, soFar);  
        }  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 1, 2 } { }  
    }  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 2 } { 1 }  
        if (elems.isEmpty()) {  
            cout << soFar << endl;  
        } else {  
            int elem = elems.first();  
            Set<int> remaining = elems - elem; { } 2 elem remaining  
            /* Option 1: Include this element. */  
            listSubsetsOf(remaining, soFar + elem);  
            /* Option 2: Exclude this element. */  
            listSubsetsOf(remaining, soFar);  
        }  
    }  
}
```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 1, 2 } { } }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { 2 } { 1 } }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) { { } { 1, 2 } }  
    elems soFar  
  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```

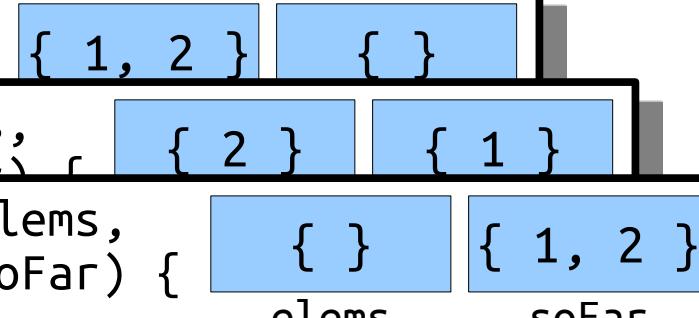
```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar);  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar);  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```



The diagram illustrates the state of two sets, `elems` and `soFar`, across three recursive calls:

- Initial call: `elems` is $\{1, 2\}$ and `soFar` is $\{\}$.
- Second call (when `elems` is $\{2\}$): `soFar` is $\{1\}$.
- Final call (when `elems` is $\{\}$): `soFar` is $\{1, 2\}$.

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar);  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        if (elems.isEmpty()) {  
            cout << soFar << endl;  
        } else {  
            int elem = elems.first();  
            Set<int> remaining = elems - elem;  
  
            /* Option 1: Include this element. */  
            listSubsetsOf(remaining, soFar + elem);  
  
            /* Option 2: Exclude this element. */  
            listSubsetsOf(remaining, soFar);  
        }  
    }  
}
```



The diagram illustrates the state of two sets, `elems` and `soFar`, across three recursive calls:

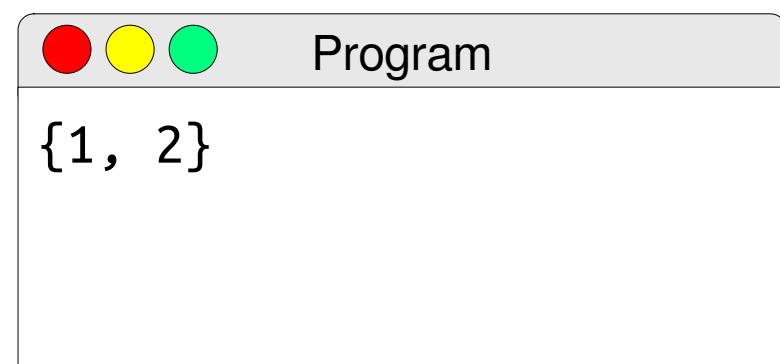
- Initial call: `elems` is `{1, 2}` and `soFar` is `{}`.
- First recursive call: `elems` is `{2}` and `soFar` is `{1}`.
- Second recursive call: `elems` is `{}` and `soFar` is `{1, 2}`.

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar);  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        if (elems.isEmpty()) {  
            cout << soFar << endl;  
        } else {  
            int elem = elems.first();  
            Set<int> remaining = elems - elem;  
  
            /* Option 1: Include this element. */  
            listSubsetsOf(remaining, soFar + elem);  
  
            /* Option 2: Exclude this element. */  
            listSubsetsOf(remaining, soFar);  
        }  
    }  
}
```



Program

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        { 1, 2 } { }  
        { 2 } { 1 }  
        { } { 1, 2 }  
    }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {  
    { } { }  
    { } { 1, 2 }  
    elems soFar  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```



```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar) {  
        { 1, 2 } { }  
        { 2 } { 1 }  
        { } { 1, 2 }  
    }  
}  
  
void listSubsetsOf(const Set<int>& elems,  
                  const Set<int>& soFar) {  
    { } { }  
    { } { 1, 2 }  
    if (elems.isEmpty()) {  
        cout << soFar << endl;  
    } else {  
        int elem = elems.first();  
        Set<int> remaining = elems - elem;  
  
        /* Option 1: Include this element. */  
        listSubsetsOf(remaining, soFar + elem);  
  
        /* Option 2: Exclude this element. */  
        listSubsetsOf(remaining, soFar);  
    }  
}
```



Program

{1, 2}

```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

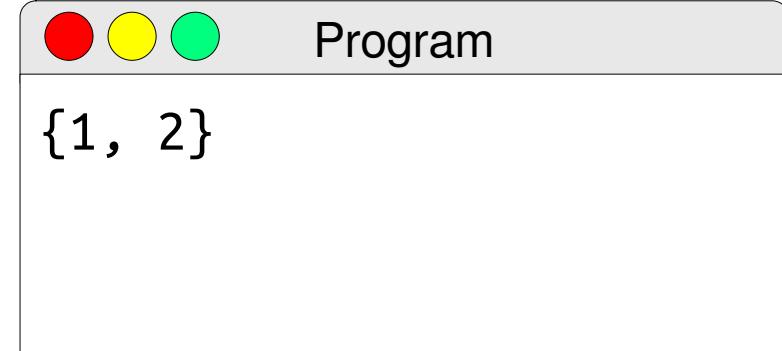
    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 2 } { 1 } }

    if (elems.isEmpty()) {
        cout << soFar << endl;
    } else {
        int elem = elems.first();
        Set<int> remaining = elems - elem; { 2 } { } elem remaining

        /* Option 1: Include this element. */
        listSubsetsOf(remaining, soFar + elem);

        /* Option 2: Exclude this element. */
        listSubsetsOf(remaining, soFar);
    }
}

```



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

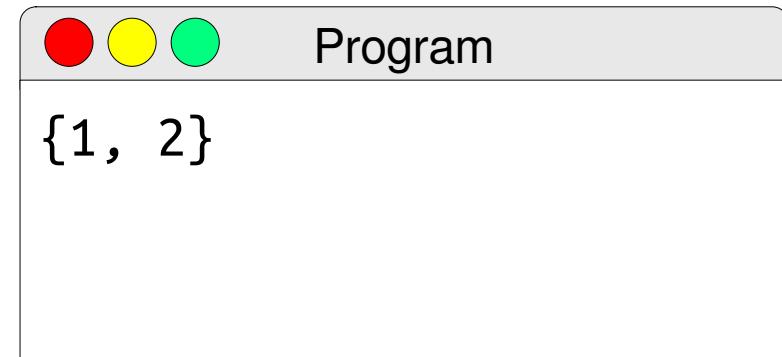
    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 2 } { 1 } }

}

if (elems.isEmpty()) {
    cout << soFar << endl;
} else {
    int elem = elems.first();
    Set<int> remaining = elems - elem; { 2 } { }
    /* Option 1: Include this element. */
    listSubsetsOf(remaining, soFar + elem);

    /* Option 2: Exclude this element. */
    listSubsetsOf(remaining, soFar);
}
}
}

```



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 1, 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { 1 }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { 1 }

        elems soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```



Program

{1, 2}

```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 1, 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { 1 }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { 1 }

        elems      soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```



Program

{1, 2}

```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 1, 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { 1 }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { 1 }

        elems      soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```



Program

{1, 2}

```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 1, 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { 1 }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { 1 }

        elems soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```



Program

```
{1, 2}
{1}
```

```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 1, 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { 1 }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { 1 }

        elems soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```



Program

```
{1, 2}
{1}
```

```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

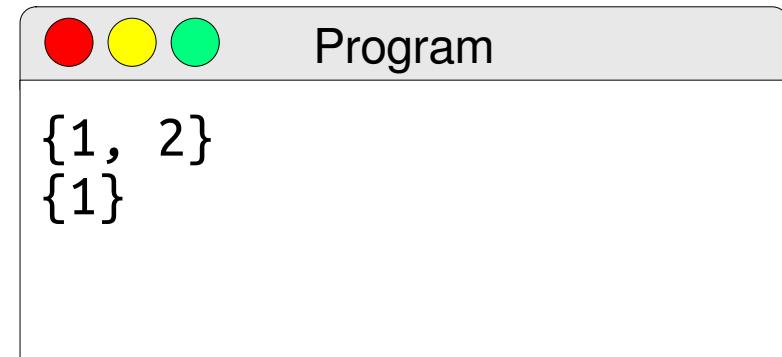
    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 2 } { 1 } }

    if (elems.isEmpty()) {
        cout << soFar << endl;
    } else {
        int elem = elems.first();
        Set<int> remaining = elems - elem; { 2 } { } elem remaining

        /* Option 1: Include this element. */
        listSubsetsOf(remaining, soFar + elem);

        /* Option 2: Exclude this element. */
        listSubsetsOf(remaining, soFar);
    }
}

```



```

int main() {

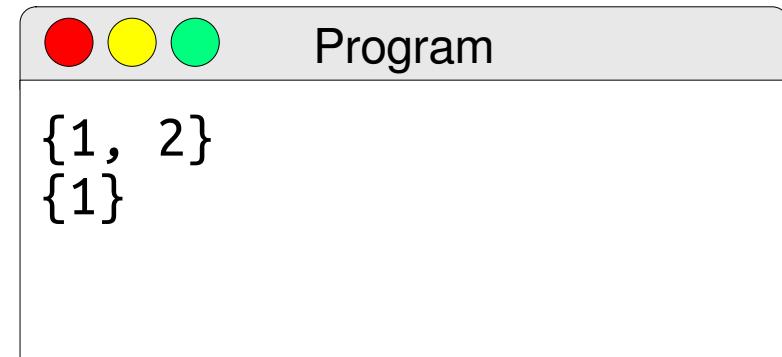
    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 2 } { 1 } }

}

if (elems.isEmpty()) {
    cout << soFar << endl;
} else {
    int elem = elems.first();
    Set<int> remaining = elems - elem; { 2 } { }
    /* Option 1: Include this element. */
    listSubsetsOf(remaining, soFar + elem);
    /* Option 2: Exclude this element. */
    listSubsetsOf(remaining, soFar);
}
}
}

```



```

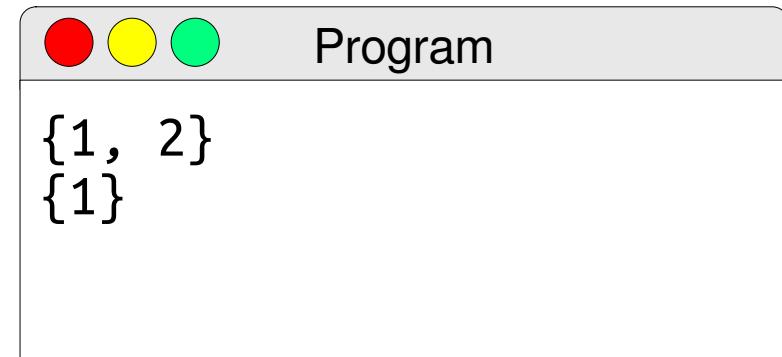
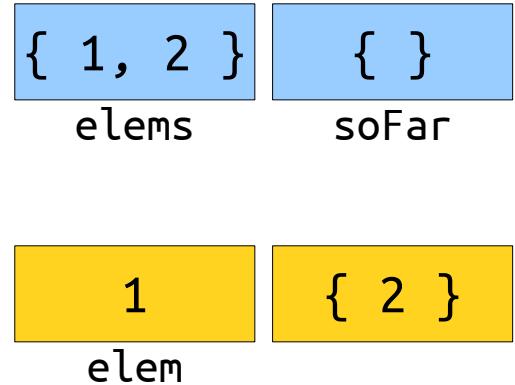
int main() {

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                      const Set<int>& soFar) {
        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
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            /* Option 1: Include this element. */
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            /* Option 2: Exclude this element. */
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        }
    }
}

```



```

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            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```

{ 1, 2 } { }

elems soFar

1 { 2 }

elem



Program

{1, 2}
{1}

```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

}

void listSubsetsOf(const Set<int>& elems,
                  const Set<int>& soFar) { { 2 } { } }

    if (elems.isEmpty()) {
        cout << soFar << endl;
    } else {
        int elem = elems.first();
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        listSubsetsOf(remaining, soFar);
    }
}
}

```



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

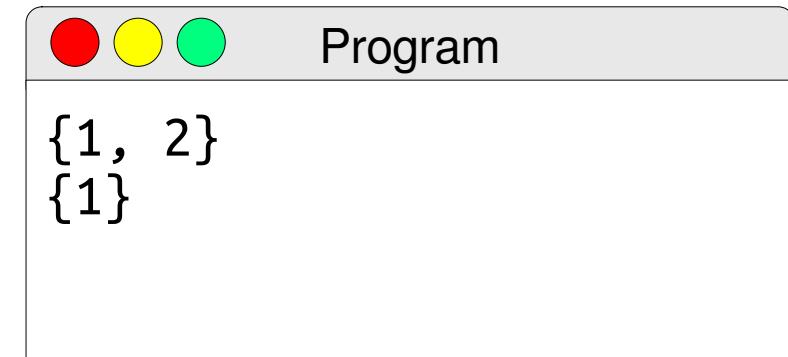
}

void listSubsetsOf(const Set<int>& elems,
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        /* Option 1: Include this element. */
        listSubsetsOf(remaining, soFar + elem);

        /* Option 2: Exclude this element. */
        listSubsetsOf(remaining, soFar);
    }
}
}

```



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

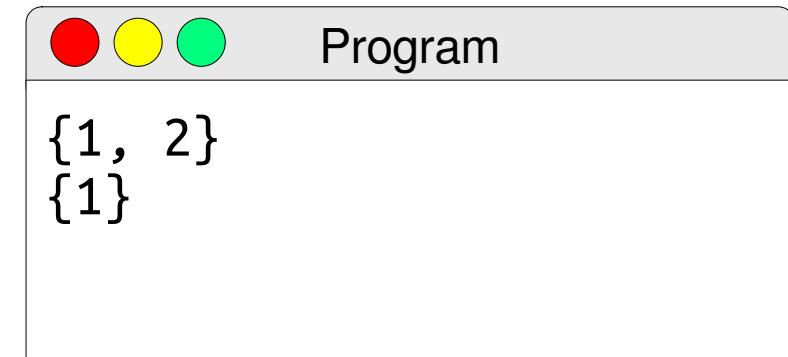
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        listSubsetsOf(remaining, soFar);
    }
}

```



```

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                      const Set<int>& soFar) { { 1, 2 } { } }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 2 } { } }

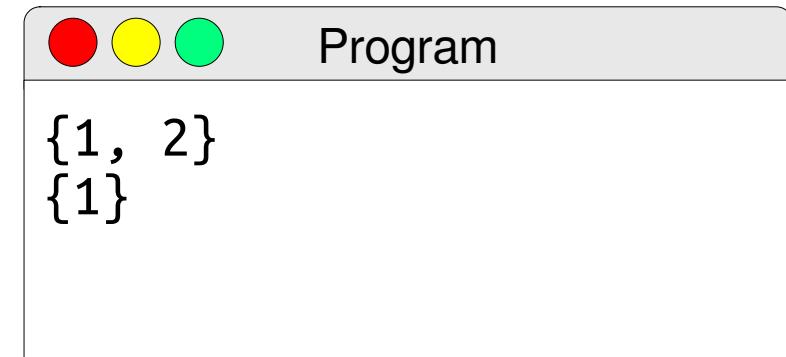
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if (elems.isEmpty()) {
    cout << soFar << endl;
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}

```



```

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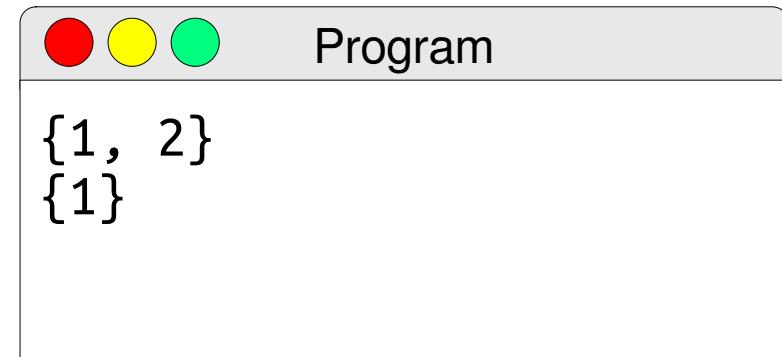
if (elems.isEmpty()) {
    cout << soFar << endl;
} else {
    int elem = elems.first(); { 2 }

    Set<int> remaining = elems - elem; elem

    /* Option 1: Include this element. */
    listSubsetsOf(remaining, soFar + elem);

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    listSubsetsOf(remaining, soFar);
}
}
}

```



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

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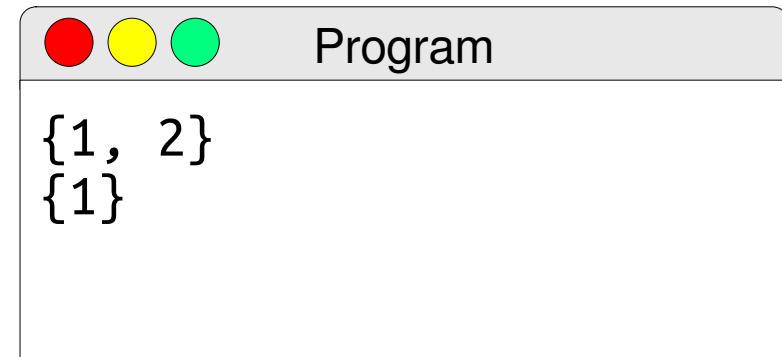
}

if (elems.isEmpty()) {
    cout << soFar << endl;
} else {
    int elem = elems.first();
    Set<int> remaining = elems - elem; 2 elem

    /* Option 1: Include this element. */
    listSubsetsOf(remaining, soFar + elem);

    /* Option 2: Exclude this element. */
    listSubsetsOf(remaining, soFar);
}
}
}

```



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 1, 2 } { }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { }

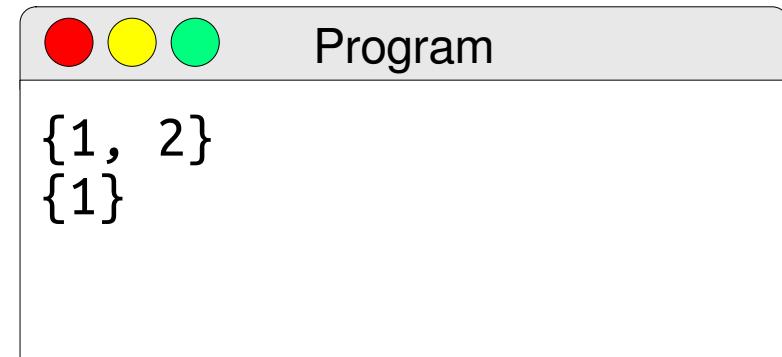
    } if (elems.isEmpty()) {
        cout << soFar << endl;
    } else {
        int elem = elems.first();
        Set<int> remaining = elems - elem;
        /* Option 1: Include this element. */
        listSubsetsOf(remaining, soFar + elem);
        /* Option 2: Exclude this element. */
        listSubsetsOf(remaining, soFar);
    }
}

```

elems soFar

2 {}

elem remaining



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 2 } { } }

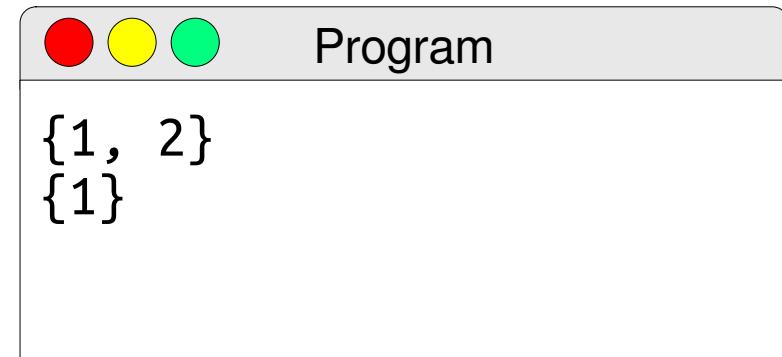
}

if (elems.isEmpty()) {
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}
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```



```

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    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { 2 }

        elems      soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
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            listSubsetsOf(remaining, soFar);
        }
    }
}

```



Program

```
{1, 2}
{1}
```

```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 1, 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { 2 }

        elems      soFar
    }

    if (elems.isEmpty()) {
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        int elem = elems.first();
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    }
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```



Program

```
{1, 2}
{1}
```

```

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    }

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                      const Set<int>& soFar) {
        { } { 2 }

        elems soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
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            int elem = elems.first();
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            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```



Program

```
{1, 2}
{1}
```

```

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                      const Set<int>& soFar) {
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                      const Set<int>& soFar) {
        { 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { 2 }

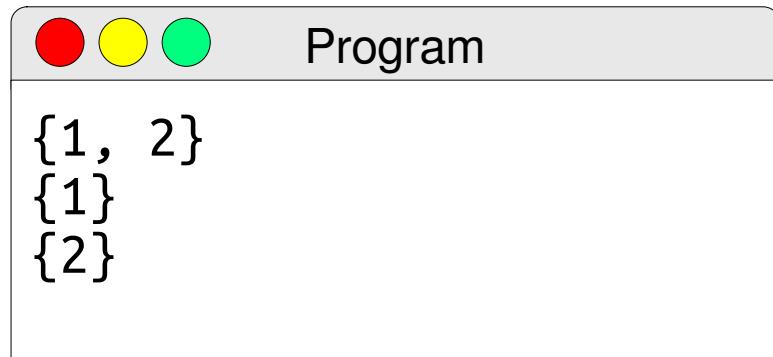
        elems soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
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            /* Option 1: Include this element. */
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}

```



```

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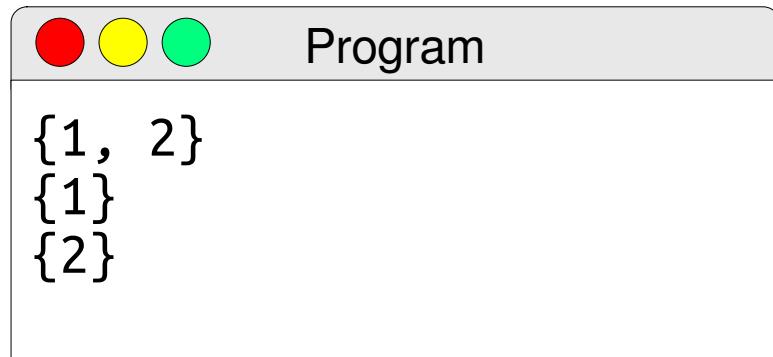
void listSubsetsOf(const Set<int>& elems,
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    if (elems.isEmpty()) {
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        listSubsetsOf(remaining, soFar);
    }
}
}
}

```



```

int main() {

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                      const Set<int>& soFar) { { 1, 2 } { } }

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                      const Set<int>& soFar) { { 2 } { } }

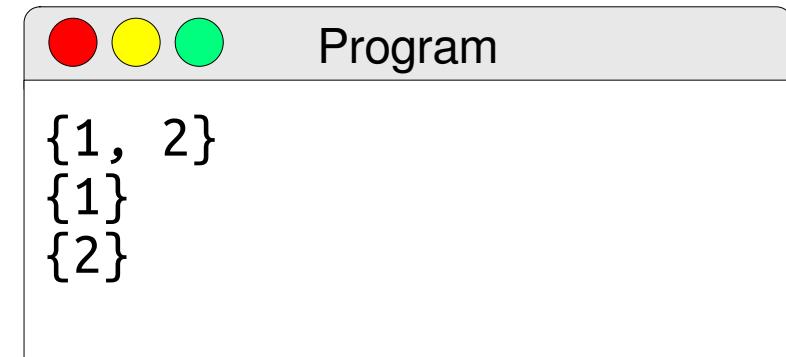
}

if (elems.isEmpty()) {
    cout << soFar << endl;
} else {
    int elem = elems.first();
    Set<int> remaining = elems - elem; { 2 } { } elem remaining

    /* Option 1: Include this element. */
    listSubsetsOf(remaining, soFar + elem);

    /* Option 2: Exclude this element. */
    listSubsetsOf(remaining, soFar);
}
}
}

```



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

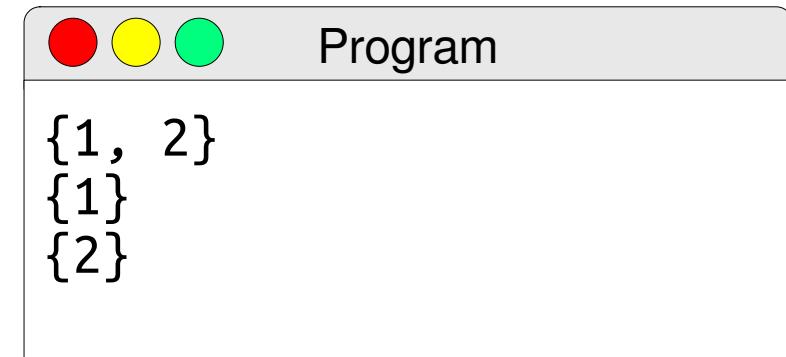
    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 2 } { } }

}

if (elems.isEmpty()) {
    cout << soFar << endl;
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    int elem = elems.first();
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```



```

int main() {

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    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
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    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { }

        elems soFar

        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
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            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```



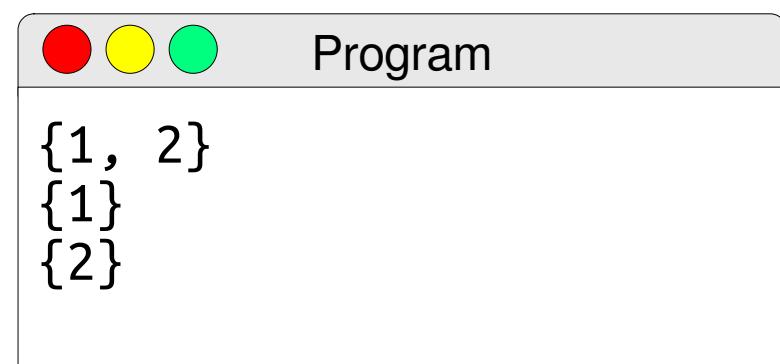
Program

```

{1, 2}
{1}
{2}

```

```
int main() {  
  
    void listSubsetsOf(const Set<int>& elems,  
                      const Set<int>& soFar);  
    { 1, 2 } { }  
}  
}  
void listSubsetsOf(const Set<int>& elems,  
                   const Set<int>& soFar);  
{ 2 } { }  
}  
void listSubsetsOf(const Set<int>& elems,  
                   const Set<int>& soFar) {  
    { } { }  
    elems soFar  
        if (elems.isEmpty()) {  
            cout << soFar << endl;  
        } else {  
            int elem = elems.first();  
            Set<int> remaining = elems - elem;  
  
            /* Option 1: Include this element. */  
            listSubsetsOf(remaining, soFar + elem);  
  
            /* Option 2: Exclude this element. */  
            listSubsetsOf(remaining, soFar);  
        }  
    }  
}
```



```

int main() {

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 1, 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { 2 } { }

    }

    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) {
        { } { }

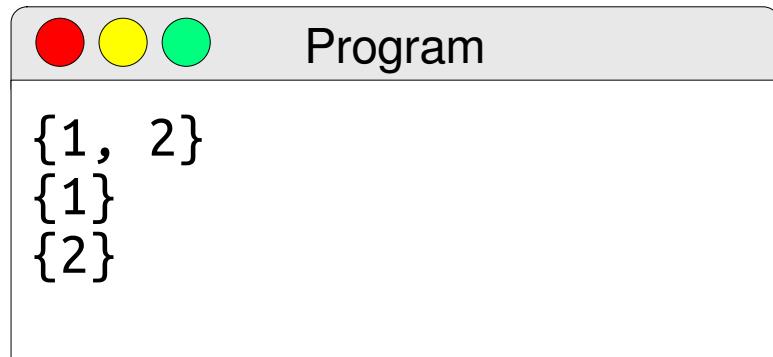
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```

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        { 2 } { }

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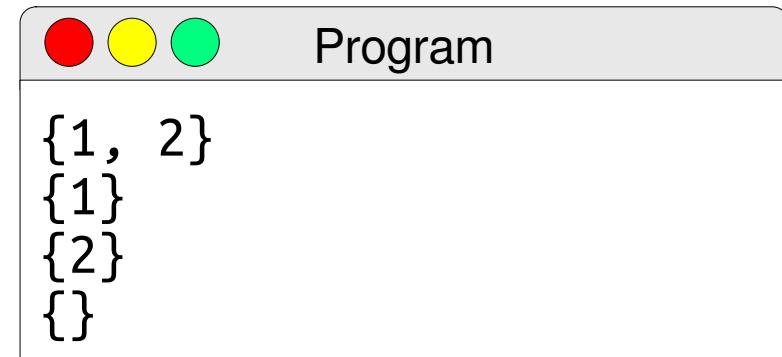
        elems soFar

        if (elems.isEmpty()) {
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```



```

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        { 2 } { }

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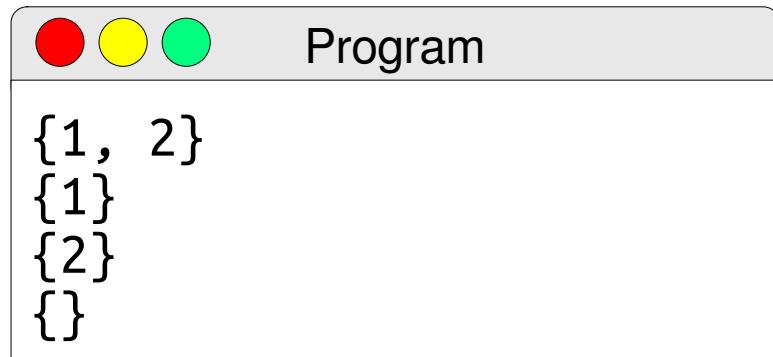
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            listSubsetsOf(remaining, soFar);
        }
    }
}

```



```

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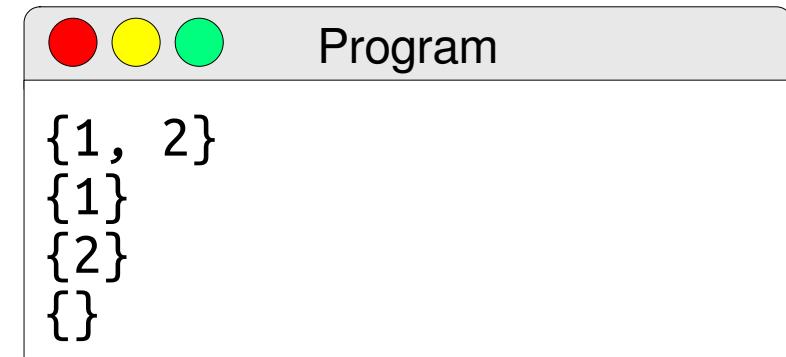
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                      const Set<int>& soFar) { { 2 } { } }

}

if (elems.isEmpty()) {
    cout << soFar << endl;
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    int elem = elems.first();
    Set<int> remaining = elems - elem; { 2 } { }
    /* Option 1: Include this element. */
    listSubsetsOf(remaining, soFar + elem);

    /* Option 2: Exclude this element. */
    listSubsetsOf(remaining, soFar);
}
}
}

```



```

int main() {

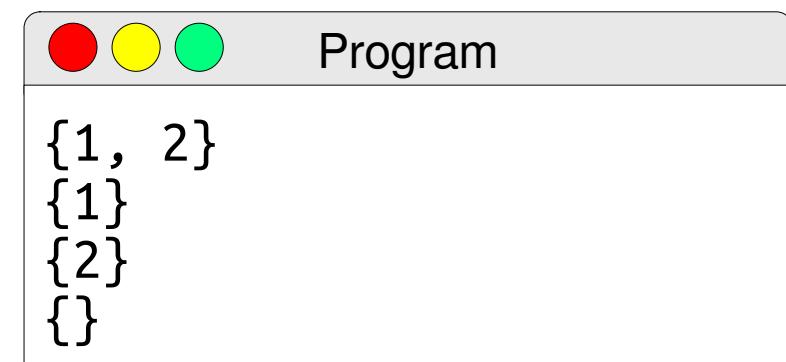
    void listSubsetsOf(const Set<int>& elems,
                      const Set<int>& soFar) { { 1, 2 } { } }

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                      const Set<int>& soFar) { { 2 } { } }

}

if (elems.isEmpty()) {
    cout << soFar << endl;
} else {
    int elem = elems.first();
    Set<int> remaining = elems - elem; { 2 } { }
    /* Option 1: Include this element. */
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    listSubsetsOf(remaining, soFar);
}
}
}

```



```

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                      const Set<int>& soFar) {
        if (elems.isEmpty()) {
            cout << soFar << endl;
        } else {
            int elem = elems.first();
            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```

{ 1, 2 }

elems

{ }

soFar

1

{ 2 }

elem



Program

{1, 2}

{1}

{2}

{}

```

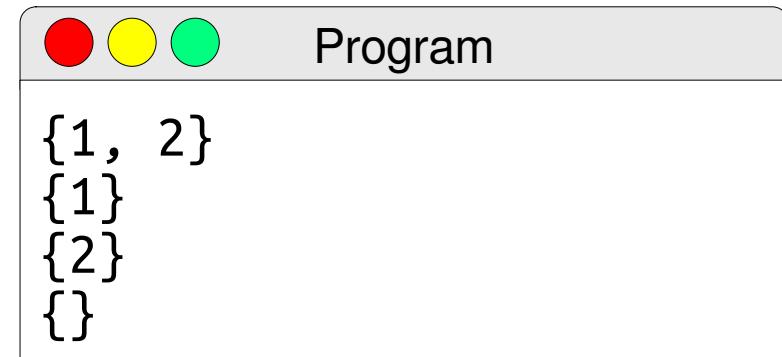
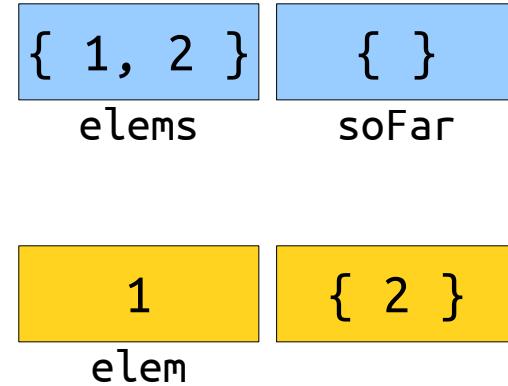
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    void listSubsetsOf(const Set<int>& elems,
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            cout << soFar << endl;
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            int elem = elems.first();
            Set<int> remaining = elems - elem;

            /* Option 1: Include this element. */
            listSubsetsOf(remaining, soFar + elem);

            /* Option 2: Exclude this element. */
            listSubsetsOf(remaining, soFar);
        }
    }
}

```



```
int main() {  
    listSubsetsOf({ 1, 2 }, {});  
    return 0;  
}
```

